SNARGs Under LWE via Propositional Proofs

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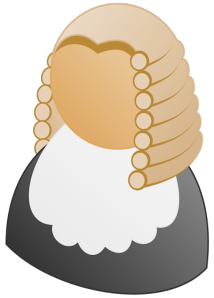
Succinct Non-Interactive Arguments (SNARGs)

CRS

CRS



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“$x \in L$”

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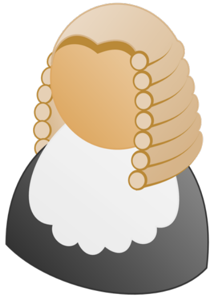

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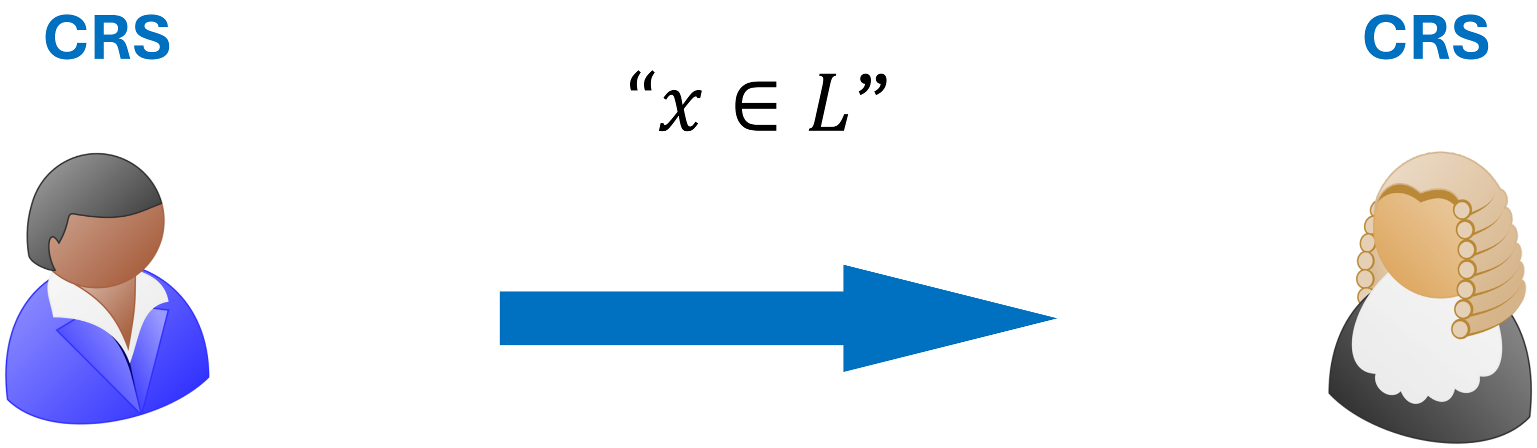

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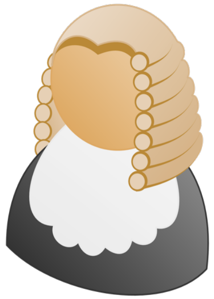
$C(x, w)$

“$x \in L$”

CRS



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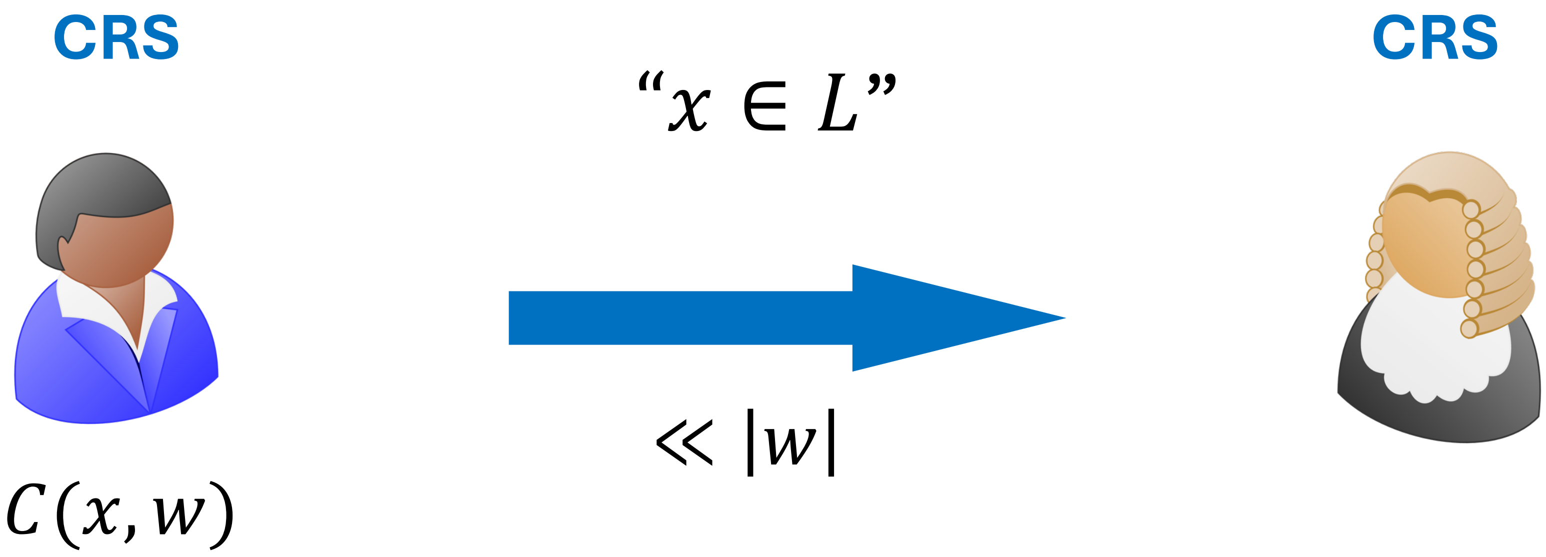

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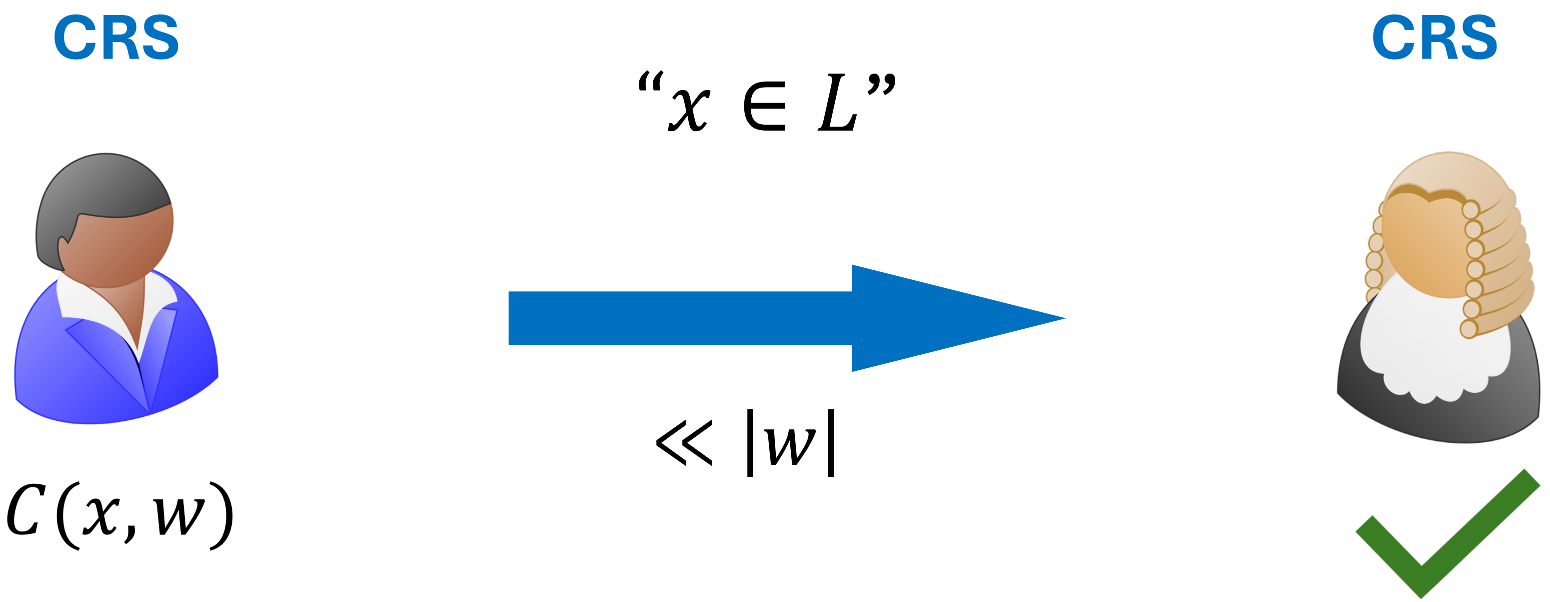

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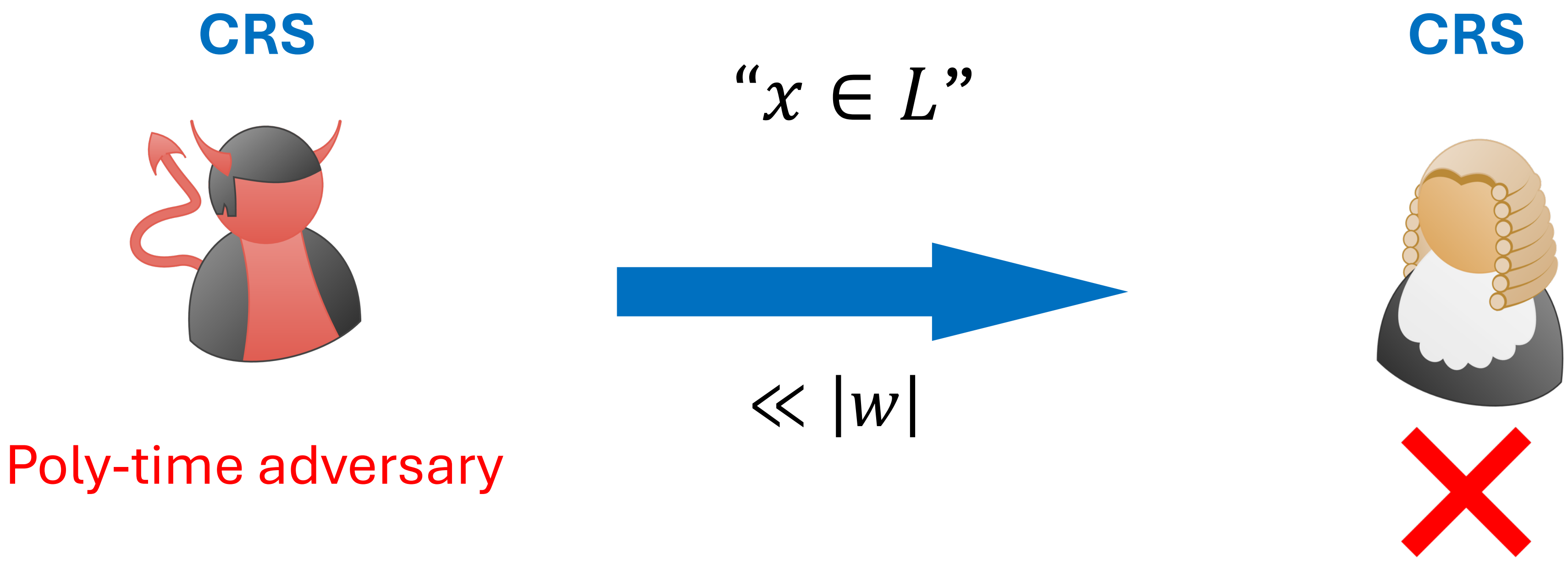

- **Completeness:** $\forall x \in L$, the honestly generated proof is accepted.
- **Soundness:** for any $x \notin L$, and any efficient adversary, the cheating proof should be rejected.

Can we build SNARGs for NP?

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Micali’00, IKO’07, GKR’08, IKOS’09, Groth’10, SBW’11, SMBW’12, Lipmaa’12, CMT’12, DFH’12, SVPBBW’12, TRMP’12, GGPR’13, BCIOP’13, BCCT’13, Thaler’13, BCGTV’13, PHGR’13, BSCGT’13, BCGGMTV’14, BCCGP’16, Groth’16, GMO’16, GLRT’17, AHIV’17, BSBCGGHPRST’17, WJBSTWW’17, BBBPWM’18, BCGMMW’18, BSBHR’18, WTSTW’18, WZCPS’18, GMNO’18, FKL’18, BBCGI’19, BBHR’19, BCRSVW’19, BSCRSVW’19, CFQ19, GWC’19, KPV’19, KPY’19, MBKM’19, Nitulescu’19, XZZPS’19, Gabizon’19, BBS’20, BSCIKS’20, BFHVXZ’20, COS’20, CHMMVW’20, KZ’20, KPPS’20, SGKS’20, SL’20, Setty’20, ZXZS’20, BMMTV’21, GLSTW’21, GMN21, GPR’21, Sta’21, ZLWZSXZ’21, Bay’22, CBBZ’22, XZCZZJBS’22, XZS’22, …

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Random Oracle Model, or Knowledge-type Assumptions

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For this Talk: A SNARG is "truly succinct" if proof size $\ll \log T$.
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Circumvent?

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Prior Works Circumvent Gentry-Wichs

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Not "Truly Succinct"

- Batch-NP [BHK17, CJJ21]
- P [KRR14, CJJ21, KVZ21]
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We don't know ***any*** truly succinct SNARGs from standard assumptions!

Can we build **truly succinct** SNARGs from standard assumptions?

(Recall: truly succinct means proof size $\ll \log T$ for any NP languages that require time T to decide.)

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(One step closer to SNARGs for NP from standard assumptions)

Our Results (I)

SNARGs from learning with errors (LWE) for NP languages that have a poly-size propositional proof of non-membership, with uniformly random CRS, where

- Proof size = poly(λ)
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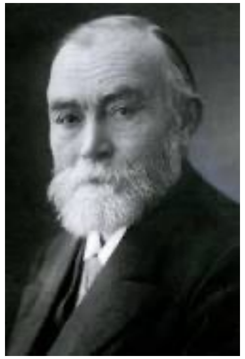
Propositional Proof of Non-Membership

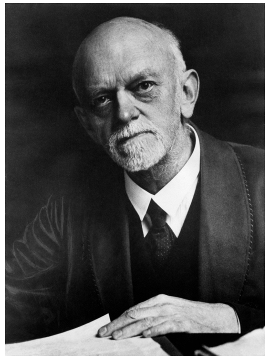
$\forall\, x \notin L$, there exists a poly-size propositional proof of

$$C(x,\cdot) = 0$$

Propositional Logic (Extended Frege)

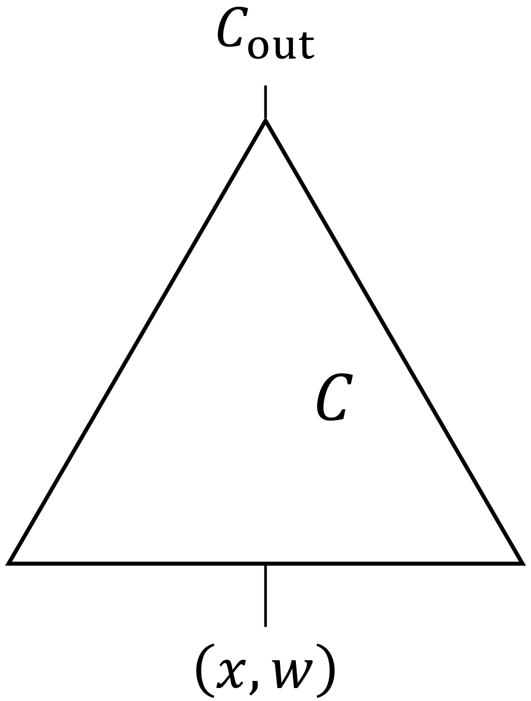


Frege



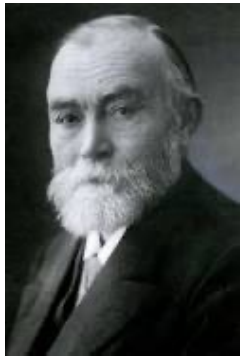
Hilbert

...



Propositional Logic (Extended Frege)

(Example: a non-membership proof of x)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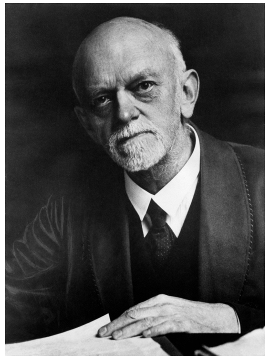

Hilbert

θ_1

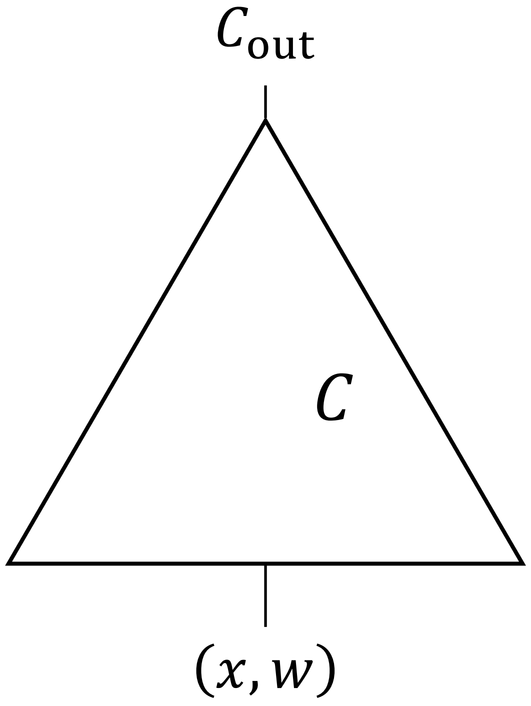
θ_2

…

θ_i

…

θ_T



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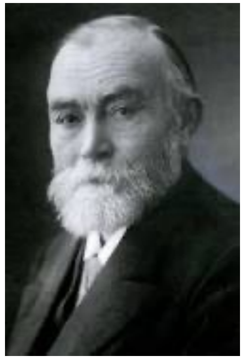

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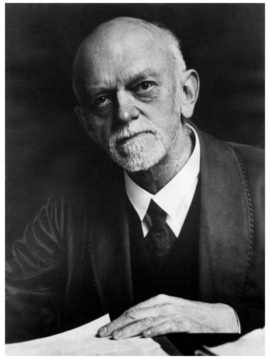


Hilbert

θ_1

"lines" in the proof: boolean formulas

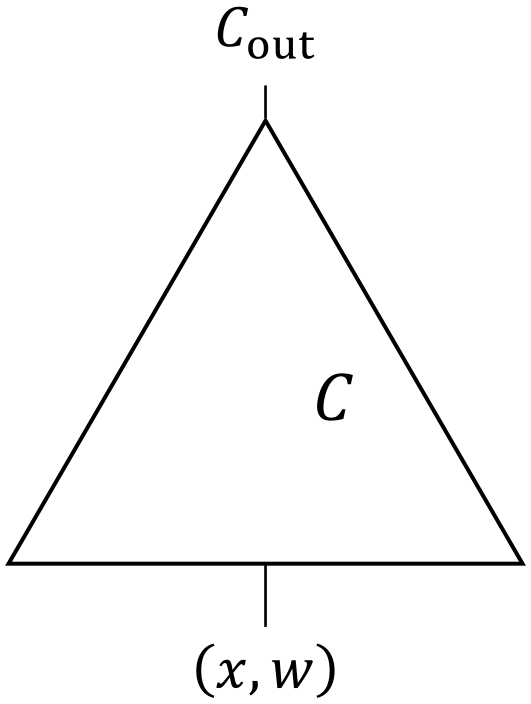
θ_2

...

θ_i

$\dots$

θ_T



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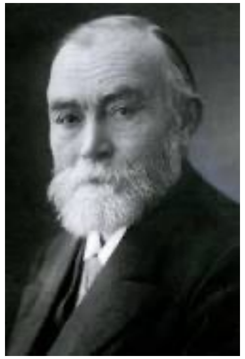

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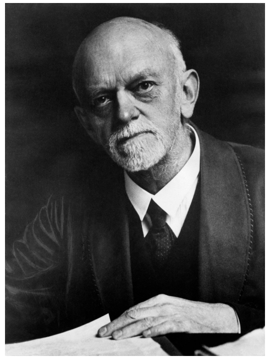

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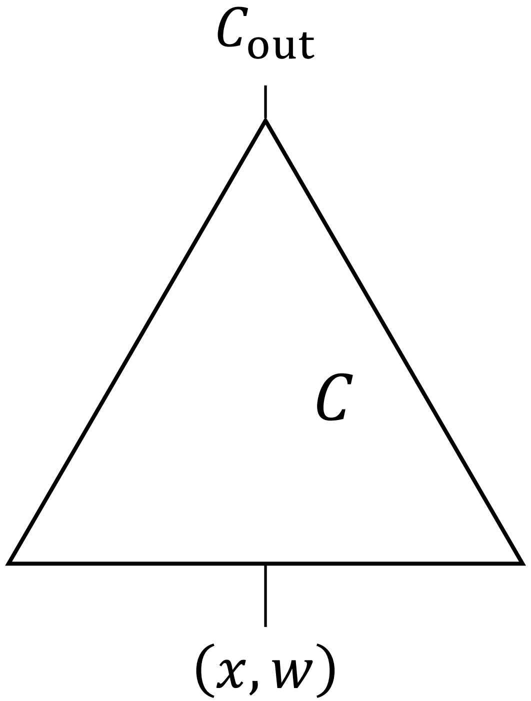

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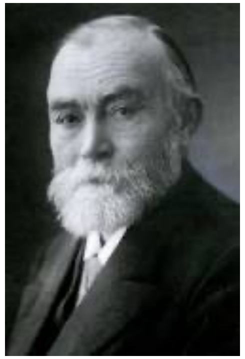

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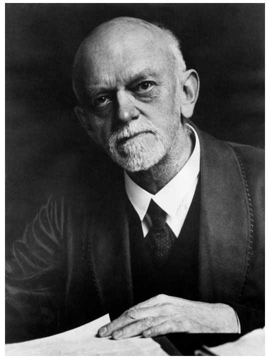

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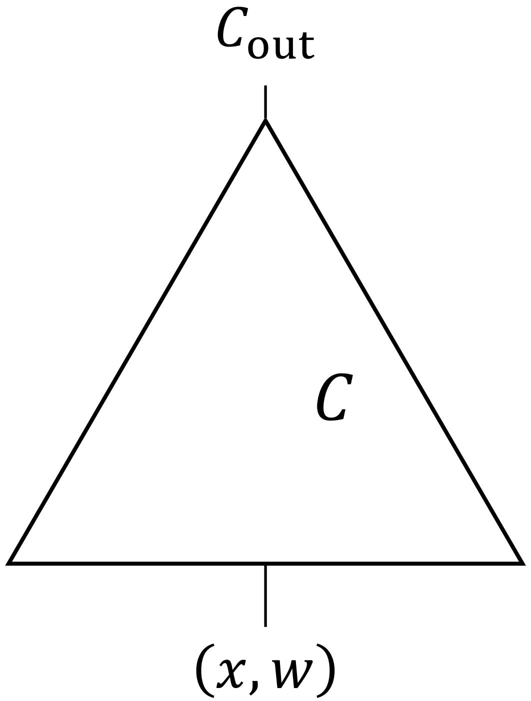

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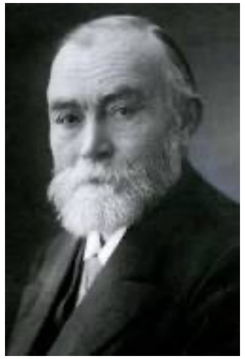

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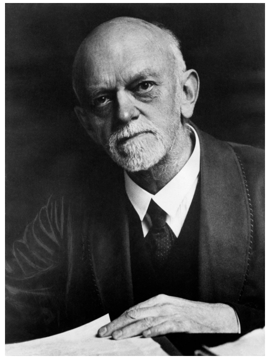

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θ_i

$$e \leftrightarrow \phi$$

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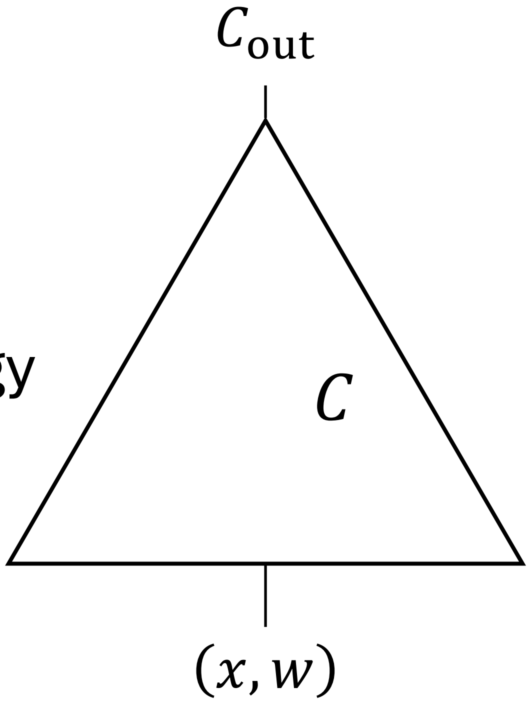

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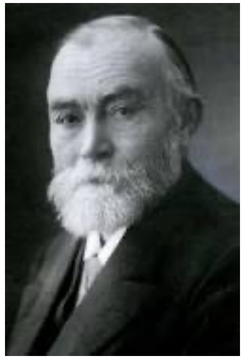

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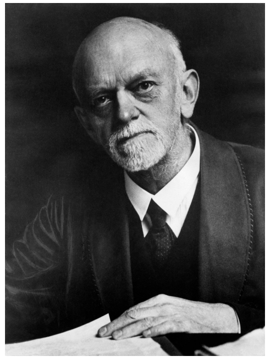

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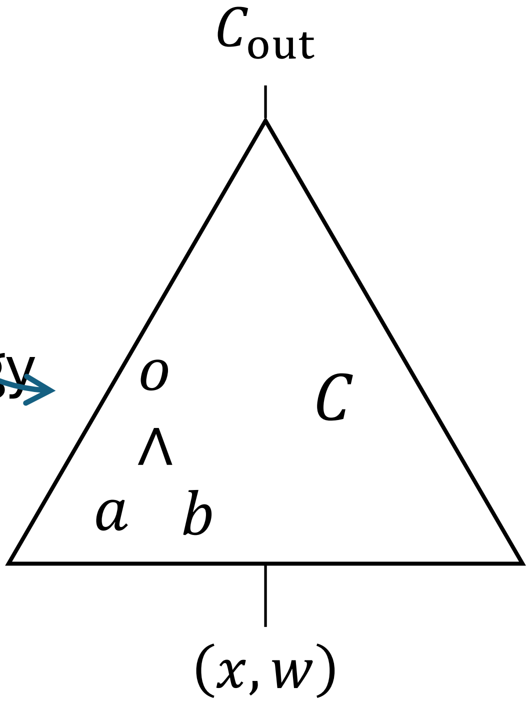

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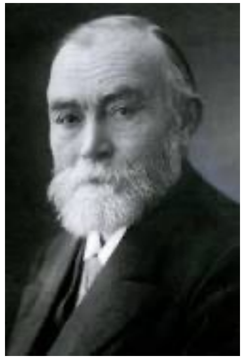

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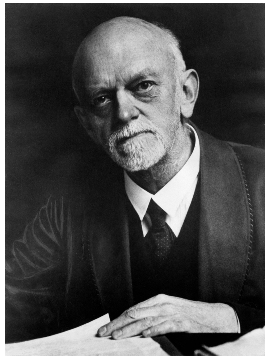

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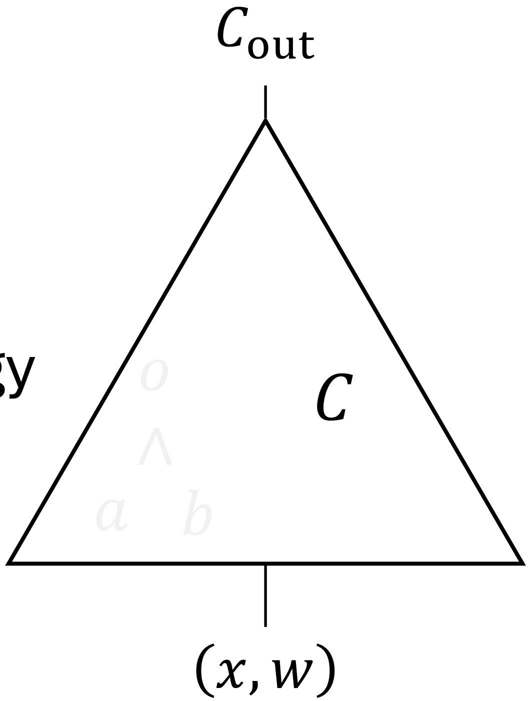

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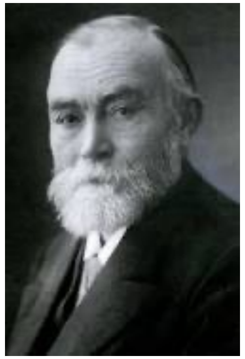

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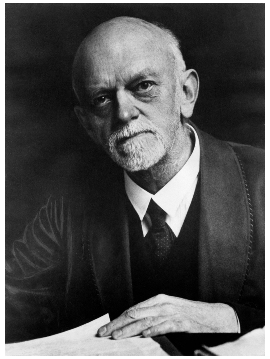

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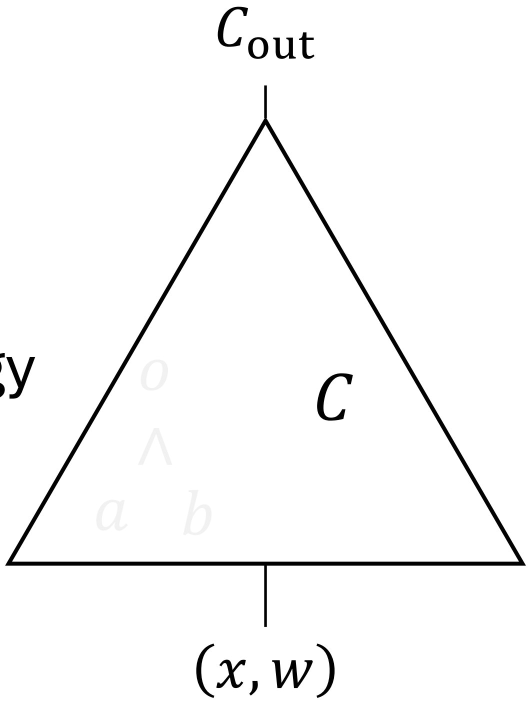

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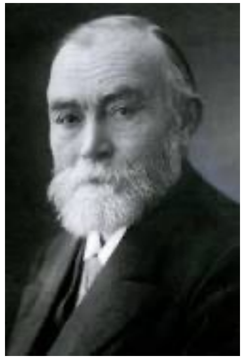

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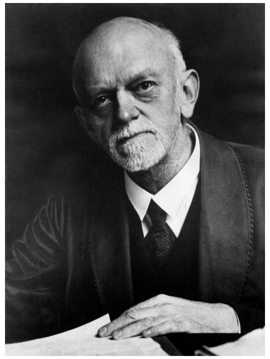

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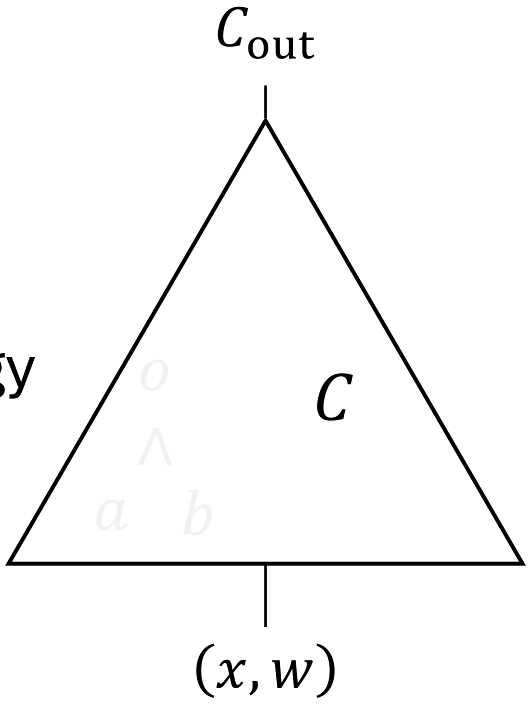

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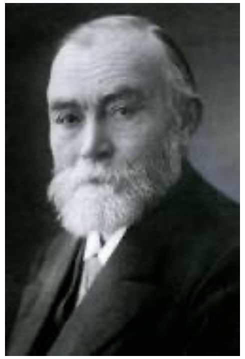

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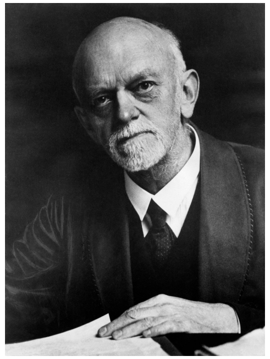

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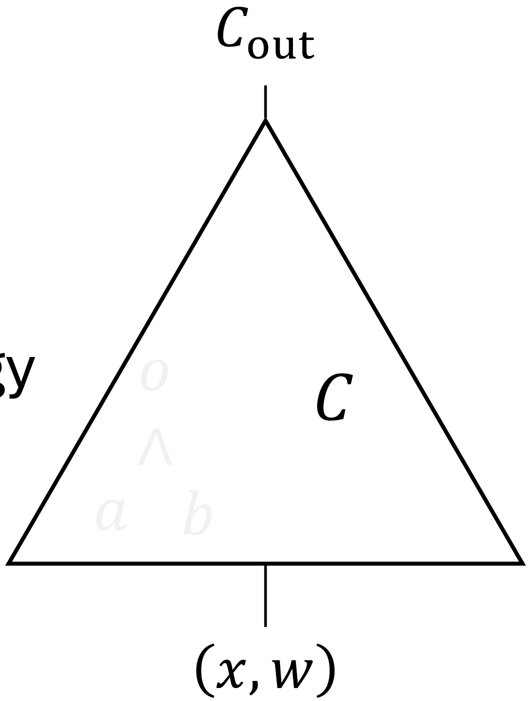

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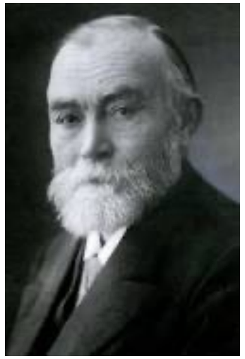

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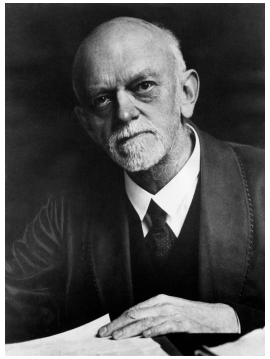

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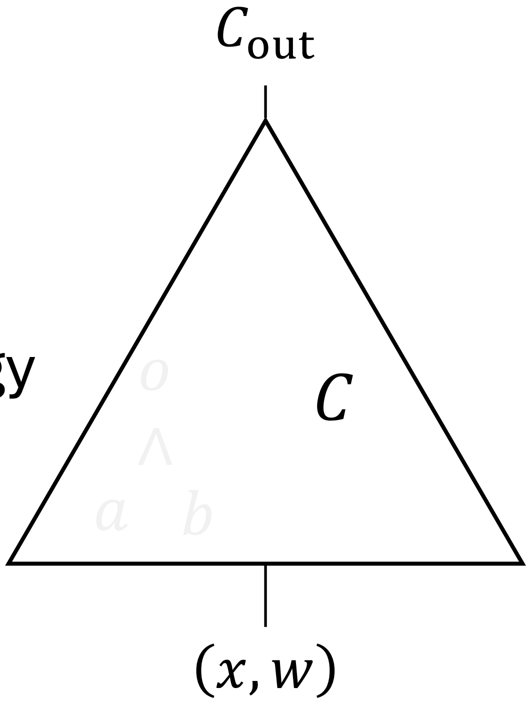

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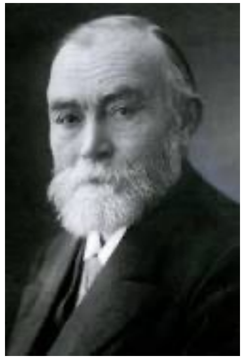
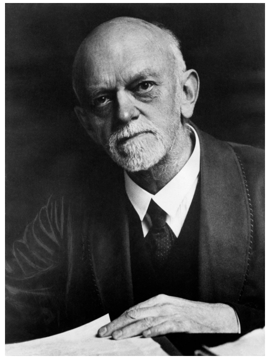


Frege



Hilbert

θ_1

"lines" in the proof: boolean formulas

θ_2

...

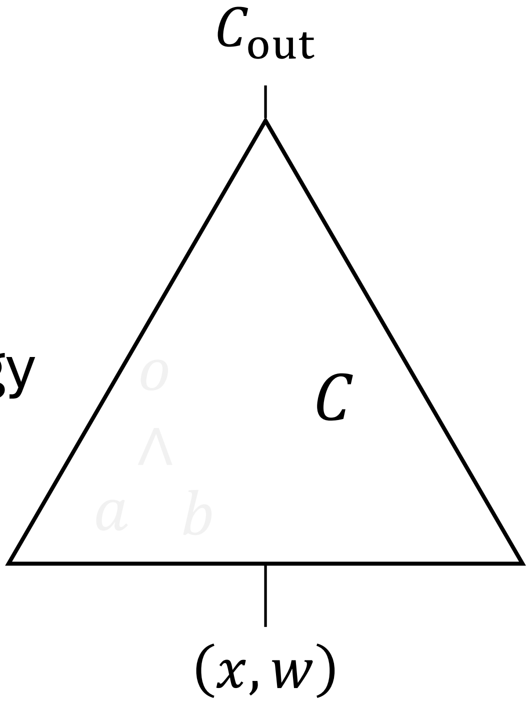
θ_i

- Premise (i.e. gates in C)
- Inference: $\theta_1' \wedge \cdots \wedge \theta_c' \to \theta_i$ is a $O(1)$-size tautology
- Extensions: define a new variable, e.g. "e"

  $e \leftrightarrow \phi$

...

θ_T



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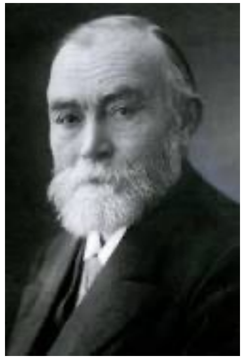

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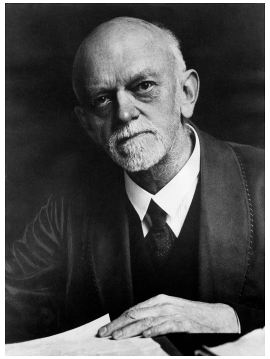

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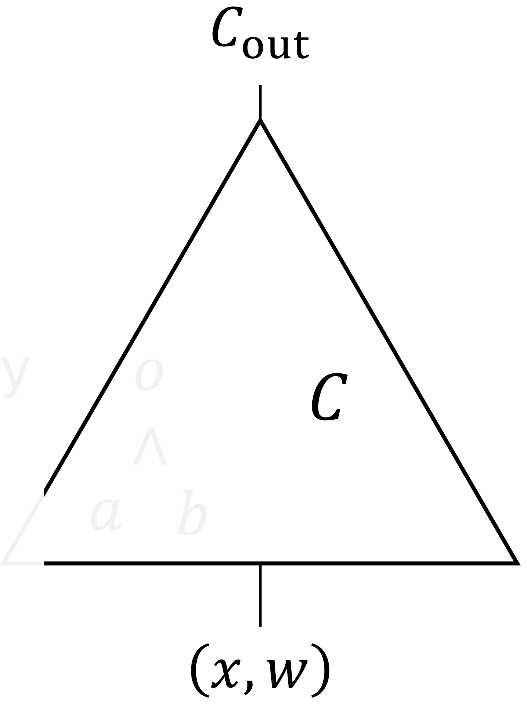
θ_T : The theorem statement itself (i.e. "$C_{\text{out}} \leftrightarrow 0$")



Recall: Our Results (I)

SNARGs from learning with errors (LWE) for NP languages that

have a **poly-size** propositional proof of non-membership, where

- Proof size = poly(λ)
- CRS size = poly(prop. proof length, λ)

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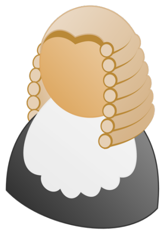
Why should I care about propositional proofs?



(a cryptographer)

Context: **Duality** between Logic and Computation

Context: **Duality** between Logic and Computation

Computation

Context: **Duality** between Logic and Computation

Logic

Computation

Context: **Duality** between Logic and Computation

Logic

Computation

Formulas

Context: **Duality** between Logic and Computation

Logic	Computation
Frege	Formulas

Context: **Duality** between Logic and Computation

Logic

Frege

Computation

Formulas

Circuits

Context: **Duality** between Logic and Computation

Logic	Computation
Frege	Formulas
Extended Frege	Circuits

Context: **Duality** between Logic and Computation

Logic	Computation
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Extended Frege	Circuits
	Poly-size Circuits

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Context: **Duality** between Logic and Computation

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Extended Frege	Circuits
Poly-size Extended Frege	Poly-size Circuits
	Poly-time Turing Machines

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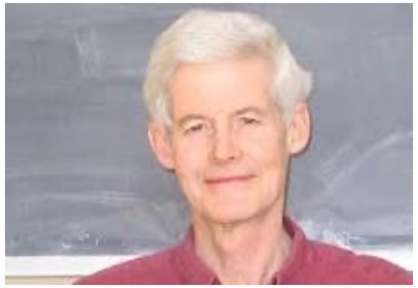 Cook's Theory PV [1975]	Poly-time Turing Machines

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Extended Frege	Circuits
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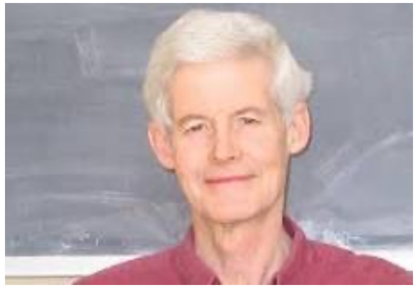
…	…



Context: **Duality** between Logic and Computation

Logic

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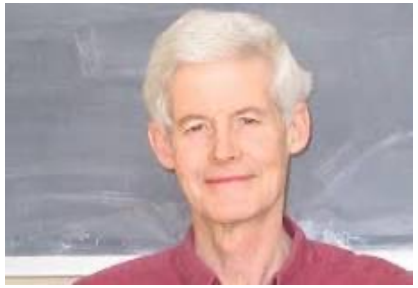

Cook's Theory PV [1975]

...

Computation

Formulas

Circuits

Computational resources (time/space) used to decide the language?

Poly-size Circuits

(Computational complexity)

Poly-time Turing Machines

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Context: **Duality** between Logic and Computation

Logic

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Resources (sizes) of proofs used to prove co-NP statements?
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Example languages?

...

Computation

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Corollary (via Cook's Theory PV)

SNARGs from LWE for decisional Diffie-Hellman language

$L = \{(g, g^a, g^b, g^{ab})\}$ over n-bits standard group.

- Proof size = poly(λ), independent of n!
- CRS size = poly(λ, n)

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$L = \{(g, g^a, g^b, g^{ab})\}$ over n-bits standard group.

- Proof size = poly(λ), independent of n!
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Fully Succinct: L requires $2^{\Omega(n^{1/3})}$ time to decide, but SNARG proof size $\ll n$.

Our Results (II)

SNARGs from T-hardness of (LWE) for NP languages that have a propositional proof of non-membership of space S and (possibly super-poly) length T, where

- |Proof| = poly($\log T$, λ)
- |CRS| = poly($S, \log T, \lambda$)

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- |Proof| = poly($\log T,\ \lambda$)
- |CRS| = poly($S, \log T, \lambda$)

"Dual" of SNARGs for bounded space non-deterministic computation NTISP(S, T)
(Proof and CRS size = poly(S, $\log T$))

Main Challenge

Recall: Indistinguishability Obfuscation (iO)

```javascript
function main() {
  console.log('hello, world');
}
main()
```

$i\mathcal{O}$ →

```javascript
function _0x19e6(_0x4d301f,_0xcaab53){var _0x3a4e72=_0x3a4e();return
_0x19e6=function(_0x19e691,_0x5809f0){_0x19e691=_0x19e691-0x14e;var
_0x16ee0b=_0x3a4e72[_0x19e691];return
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['log','199381NCGrSa','2328491tAiNSg','18mVqyqS','4cVQTsk','6PuGzwR','107410
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_0x3f0a9d;};return _0x3a4e();}(function(_0x3d9e47,_0x360e03){var
_0x3afd0b=_0x19e6,_0x2928d3=_0x3d9e47();while(!![]){try{var _0x33cc3a=-
parseInt(_0x3afd0b(0x15a))/0x1*(-parseInt(_0x3afd0b(0x158))/0x2)+-
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parseInt(_0x3afd0b(0x152))/0x5+parseInt(_0x3afd0b(0x150))/0x6*
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- **Preserve Functionality:** $iO(C)$ preserves the functionality of C
- **Indistinguishability Security:** for any C_0, C_1 that compute the same function,

$$iO(C_0) \approx_c iO(C_1)$$

Starting Point: SW-SNARGs from iO

CRS

$$\widetilde{PK} = iO\left(\begin{array}{l} PK(x,w): \\ \quad \text{if } C(x,w) = 1: \\ \quad\quad \text{output } PRF_k(x) \end{array}\right)$$

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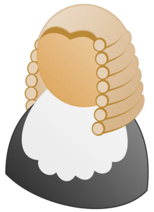

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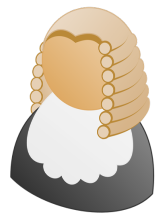
(x, w)

σ



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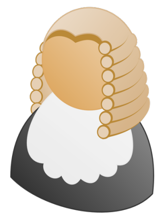

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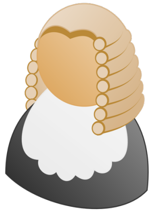

Intuition for Soundness: $\forall\, x^* \notin L$, $\widetilde{PK}$ never outputs $PRF_k(x^*)$

$\Rightarrow$ iO hides $PRF_k(x^*)$

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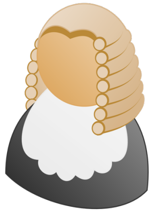
Can we instantiate this template from LWE?

(x, w)

σ

σ



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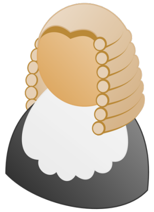
Towards SNARGs from LWE: Replace iO with FHE

(FHE: fully homomorphic encryption)



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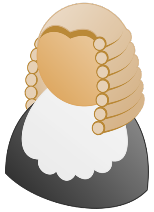

sk

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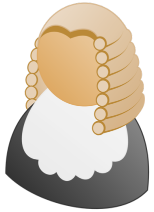
CRS

$$\mathrm{FHE}\left[\; C(x,\cdot) \;\right]$$

Eval. on w



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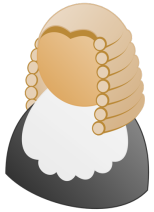
Eval. on w

$$\mathrm{FHE}\big(C(x,w)\big)$$



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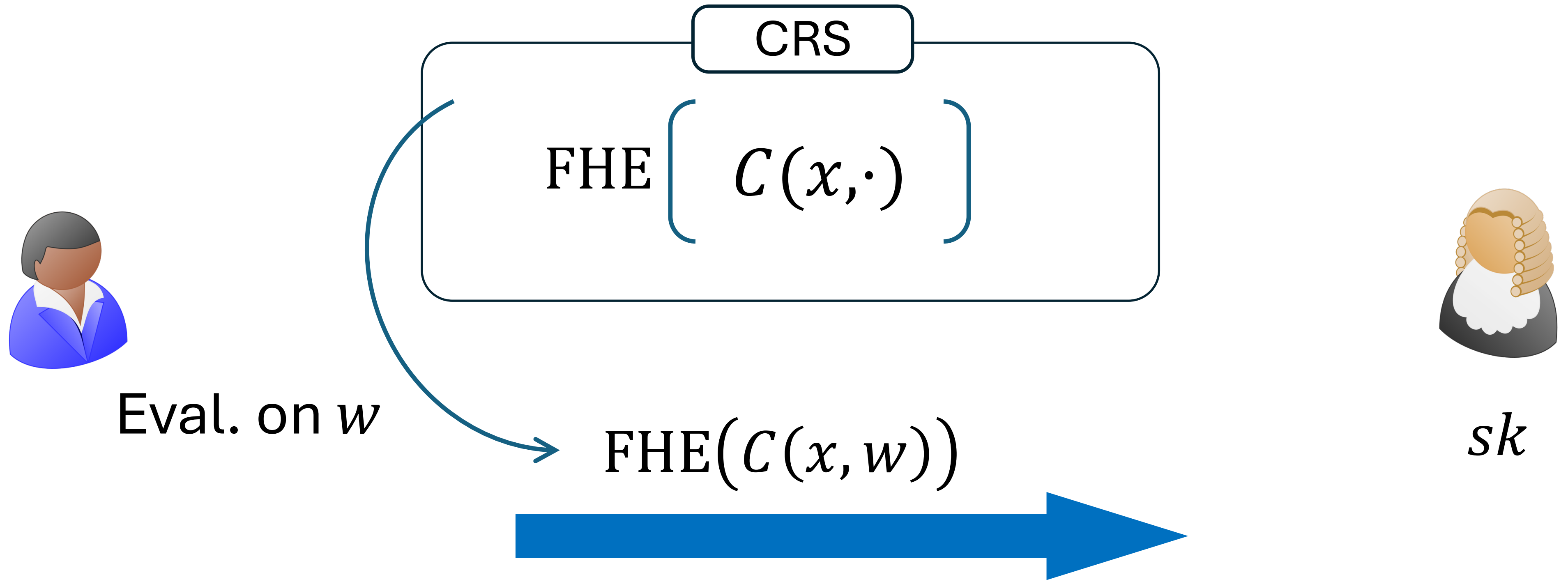

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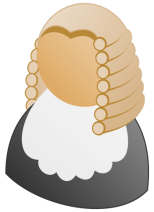

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Dec and check whether it's 1

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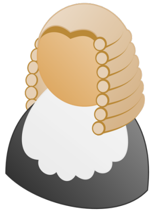
Eval. on w

$\mathrm{FHE}\big(C(x,w)\big)$ π



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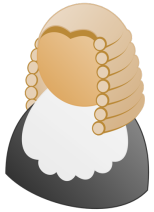

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π proves "$\mathrm{FHE}\big(C(x,w)\big)$ is computed correctly from some w."

Towards SNARGs from LWE: Replace iO with FHE

(FHE: fully homomorphic encryption)

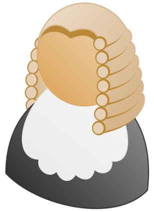
CRS

$\mathrm{FHE}\big(C(x,\cdot)\big)$

Eval. on w

We need a SNARG!

$\mathrm{FHE}\big(C(x,w)\big)$ π



sk

Dec and check whether it's 1

π proves "$\mathrm{FHE}\big(C(x,w)\big)$ is computed correctly from some w."

Towards SNARGs from LWE: Replace iO with FHE

(FHE: fully homomorphic encryption)

- CRS depends on x
- Designated Verifier

(will solve later)

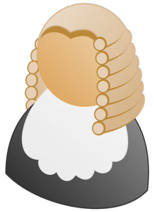
CRS

$\mathrm{FHE}\left(C(x,\cdot) \right)$

Eval. on w

$\mathrm{FHE}\big(C(x,w)\big)$ π

We need a SNARG!



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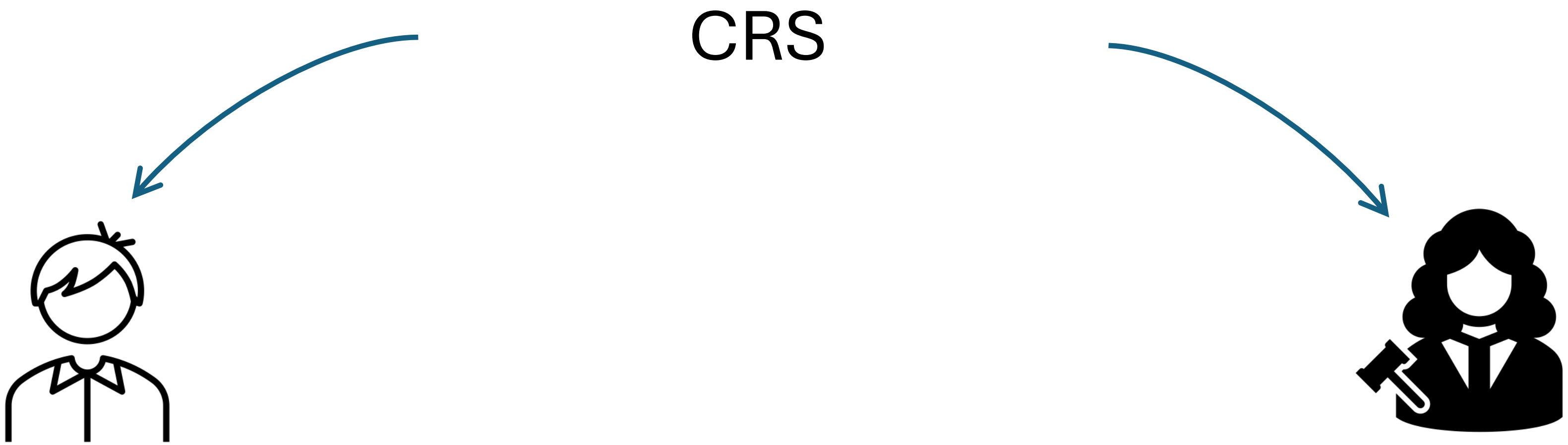
Recall: BARGs (Batch Arguments)[BHK17,CJJ21]



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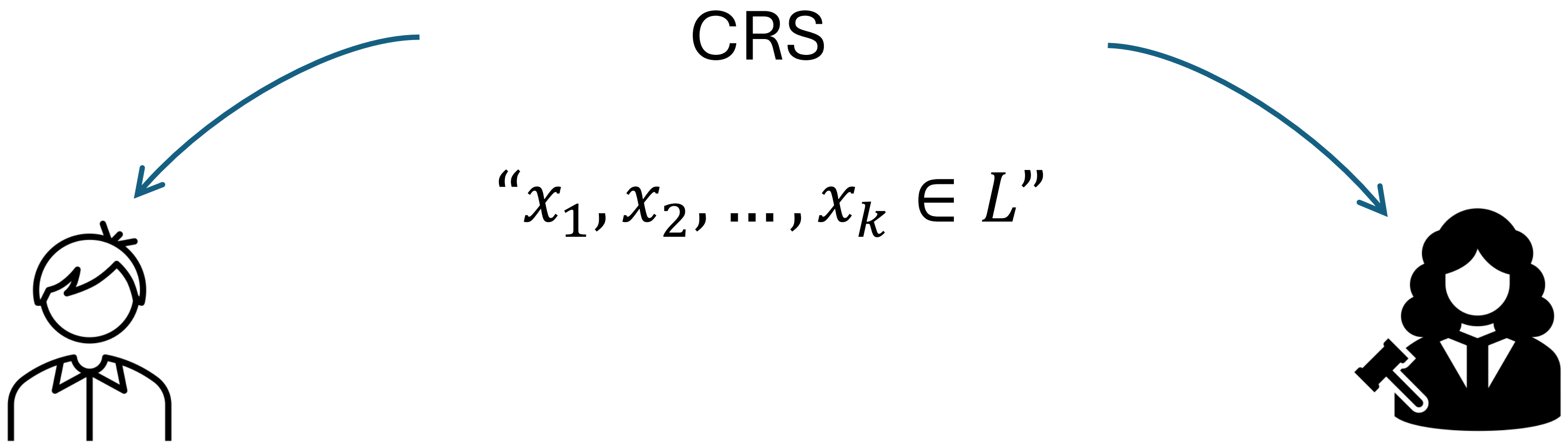

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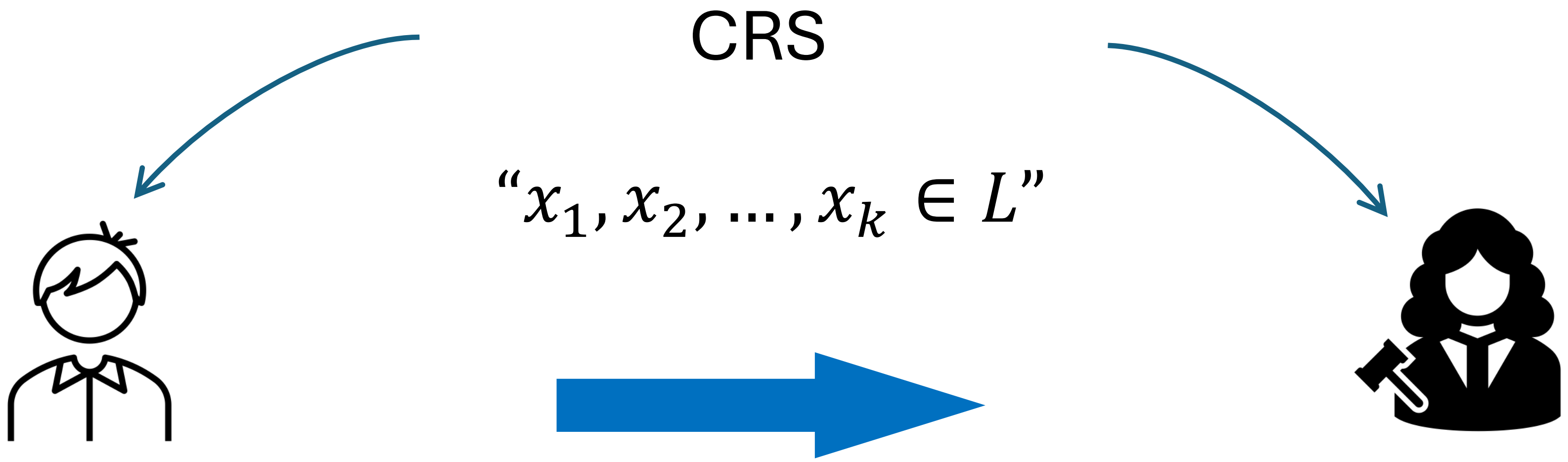

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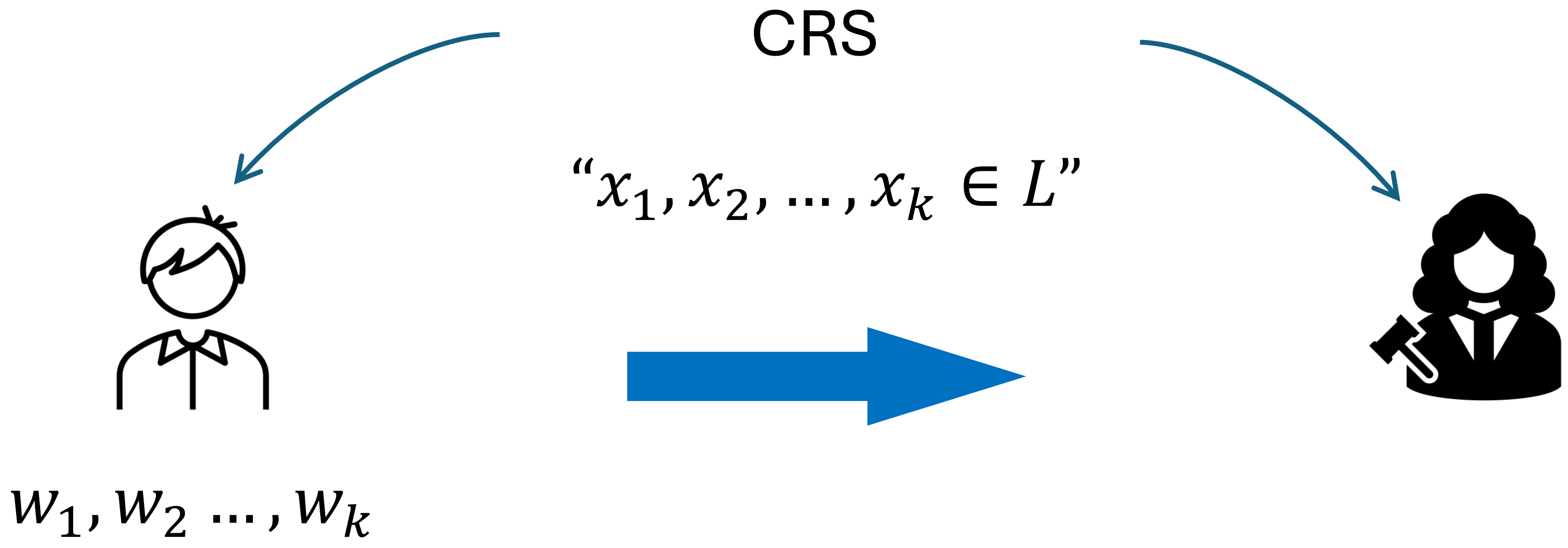
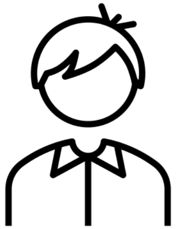


Recall: BARGs (Batch Arguments)[BHK17,CJJ21]

CRS

“$x_1, x_2, \ldots, x_k \in L$”

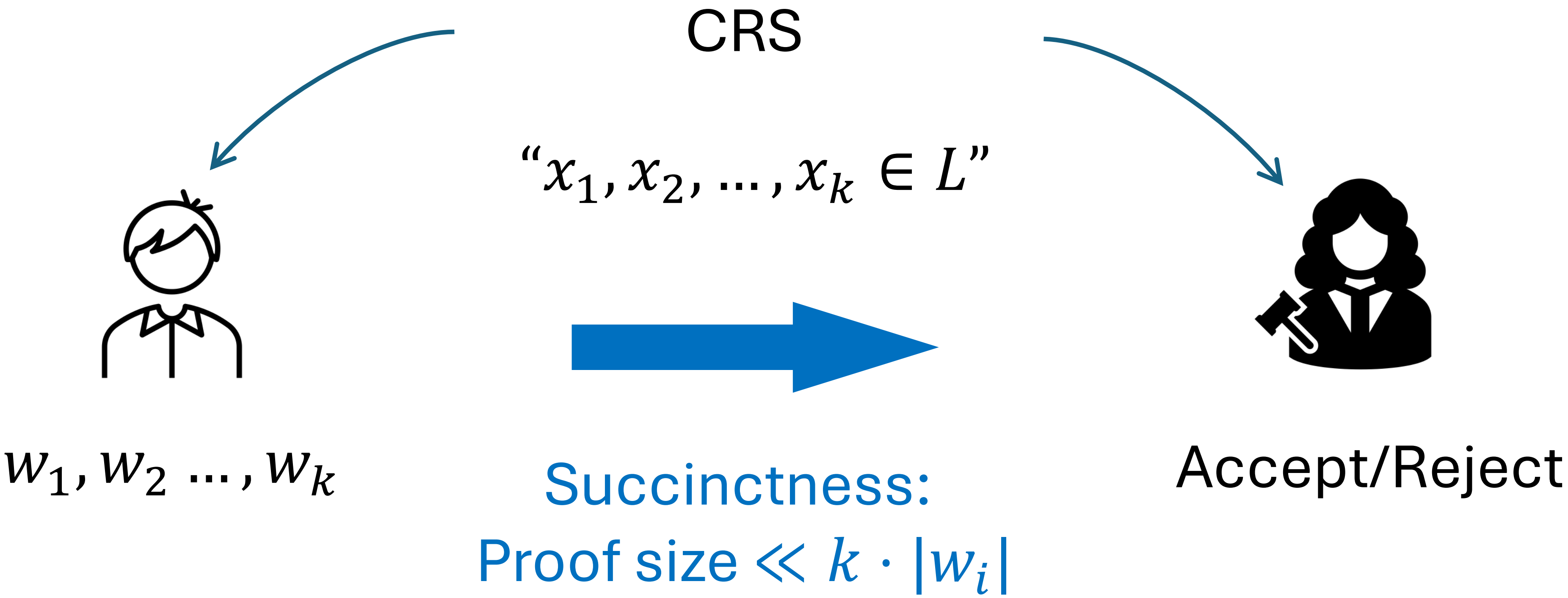


$w_1, w_2 \ldots, w_k$

Succinctness:

Proof size $\ll k \cdot |w_i|$

Recall: BARGs (Batch Arguments)[BHK17,CJJ21]



Recall: BARGs (Batch Arguments)[BHK17,CJJ21]

$\mathsf{CRS}(S) \approx_c U$

S: a **small** subset of $[k]$

Unbounded

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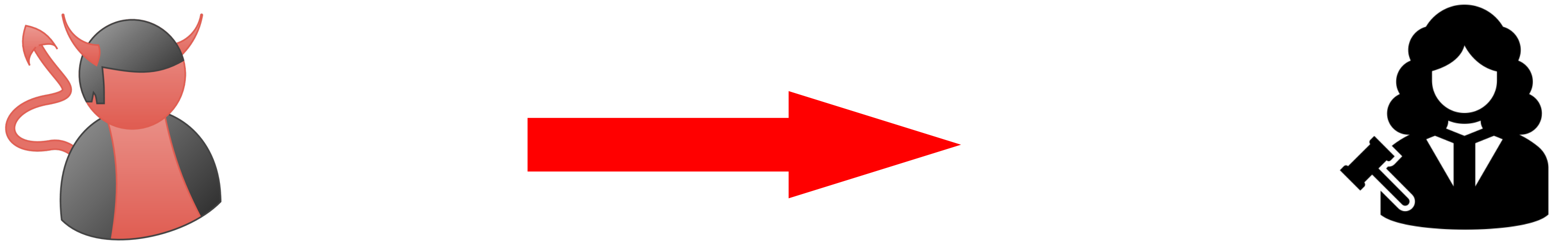

Unbounded

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Unbounded

Proof size $\approx |S| \cdot |w_i|$

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$\text{CRS}(S) \approx_c U$

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Unbounded

Somewhere Statistical Soundness

If one of the instances in S is false, then any **unbounded-time** computed cheating proof should be rejected.

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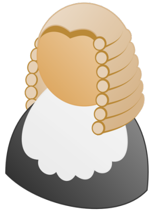
Applying BARGs

$$\text{CRS} = \text{FHE}\left[\, C(x,\cdot) \,\right]$$

$$\text{FHE}\big(C(x,w)\big)$$



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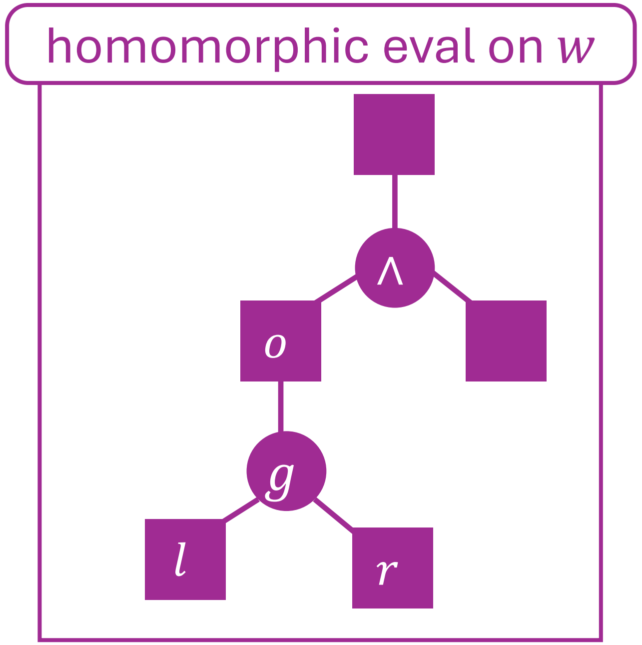

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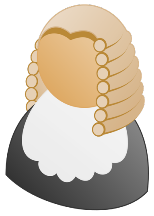
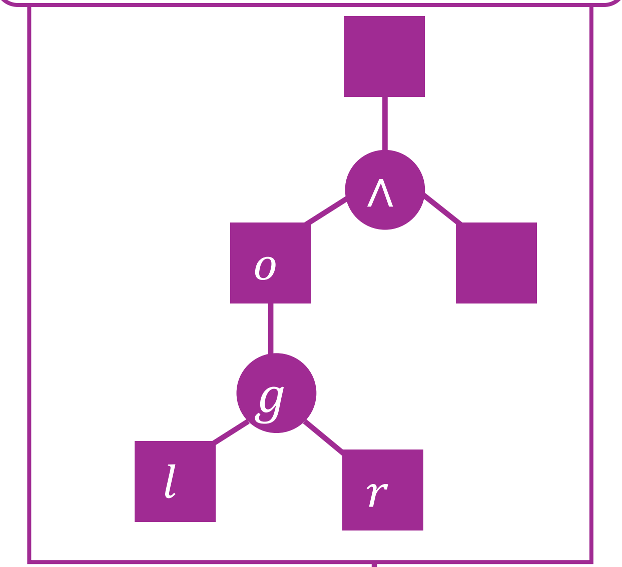


Applying BARGs

homomorphic eval on w



Merkle hash
ciphertexts of wires
(consistency)

h

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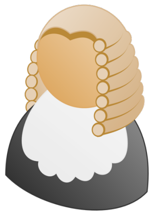

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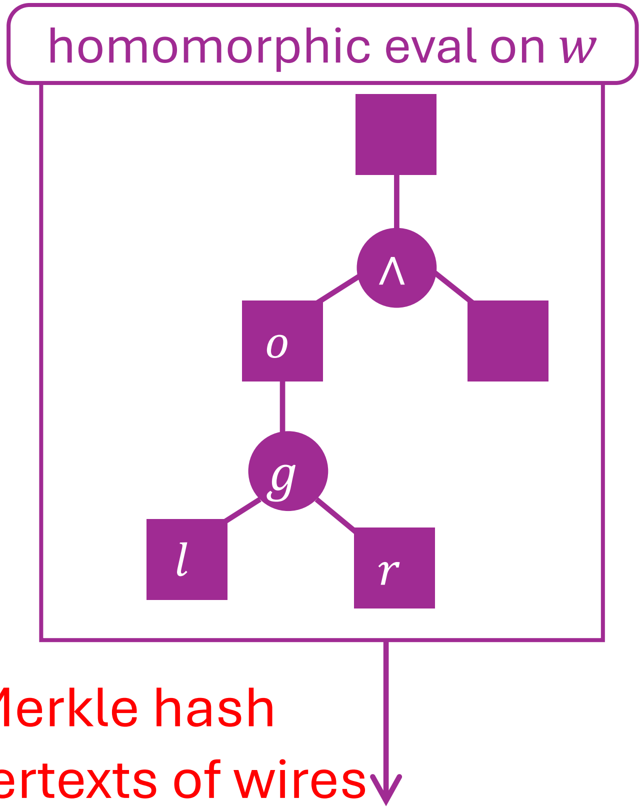

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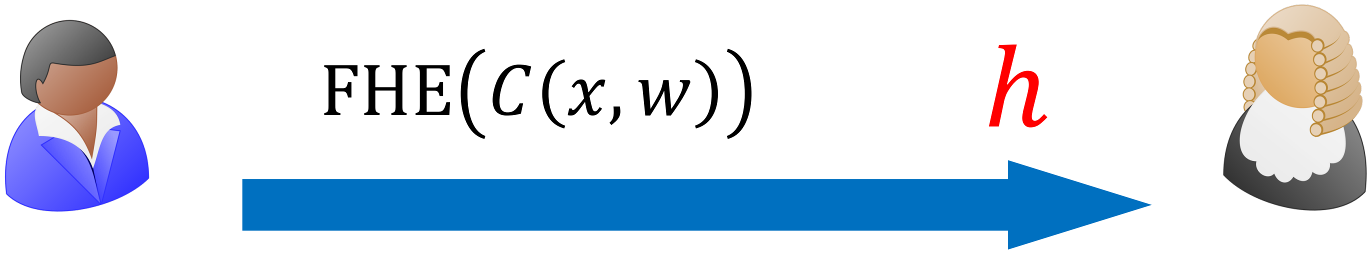

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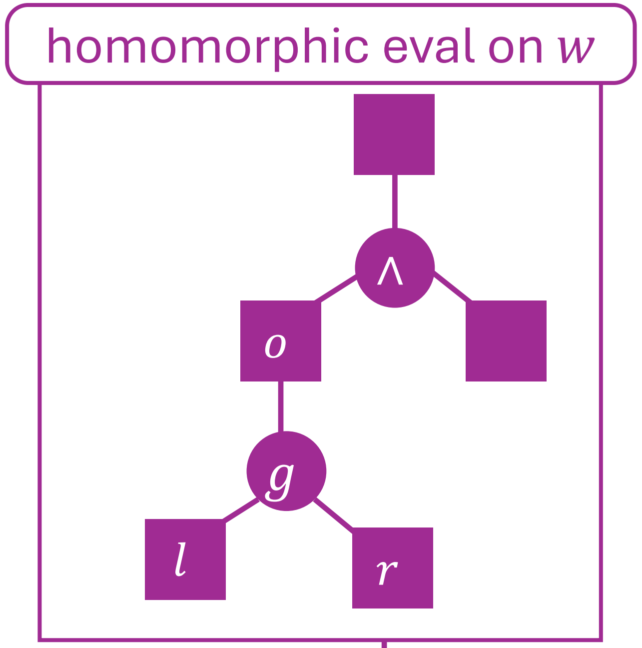

Merkle hash
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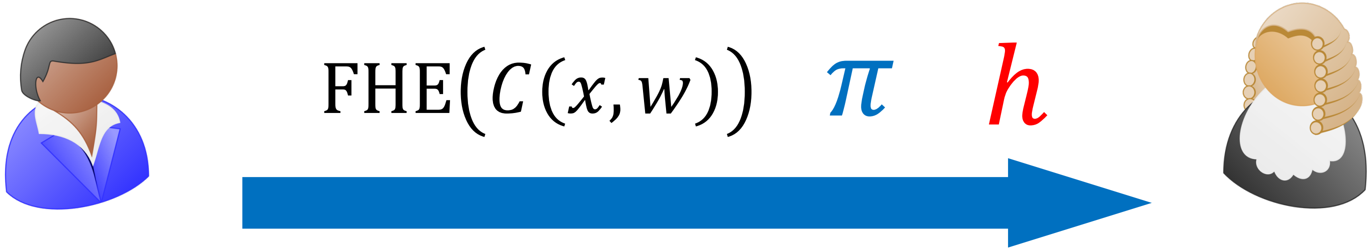
$\text{FHE}\big(C(x,w)\big) \quad \pi \quad h$



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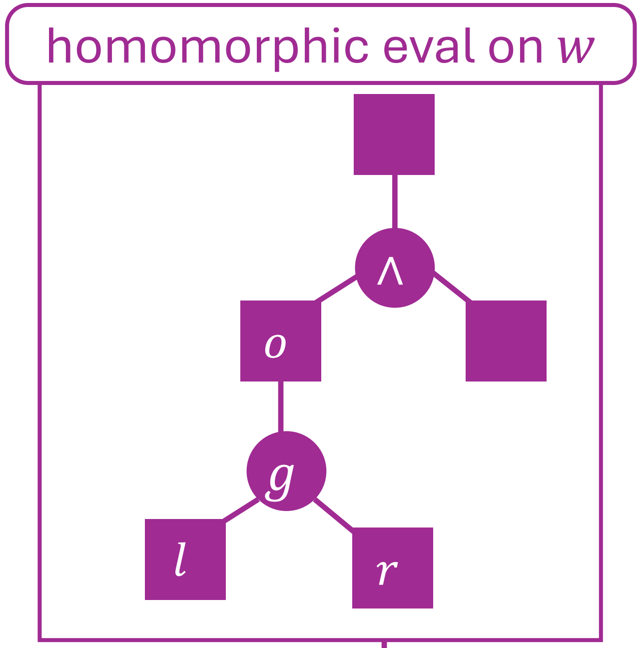

Merkle hash ciphertexts of wires (consistency)

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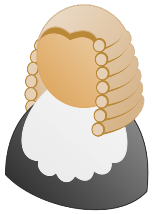
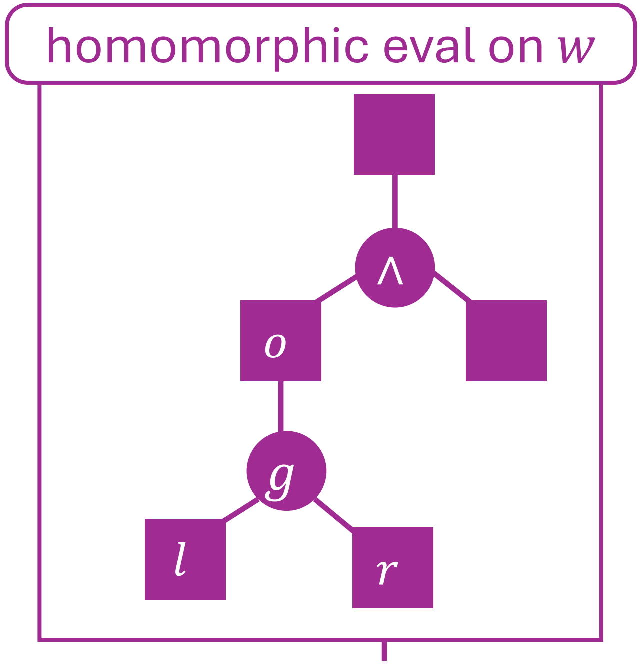
$\text{CRS} = \text{FHE}\left[C(x,\cdot) \right]$

$\text{FHE}(C(x,w))$ π h



π proves "$\forall$ gate g, $\exists$ local openings ρ_l, ρ_r, ρ_o of ct_l, ct_r, ct_o w.r.t. h, and $ct_o = g(ct_l, ct_r)$" using BARG.

Applying BARGs



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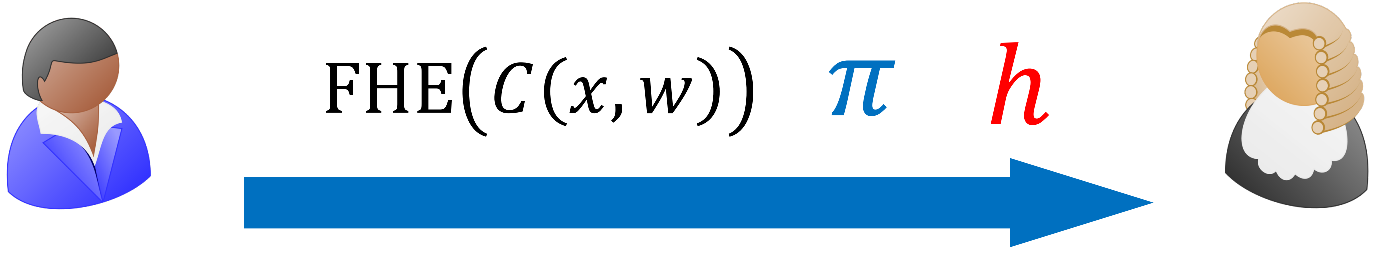

How to Prove the Soundness?

BARG $\Rightarrow$ only part of the evaluation is correct

(An Informal) **Barrier** in Soundness Reduction

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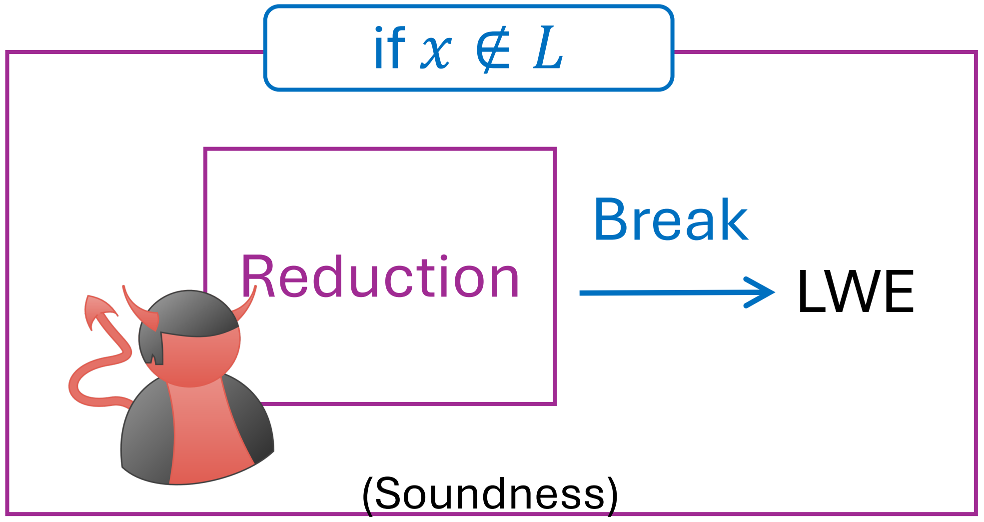

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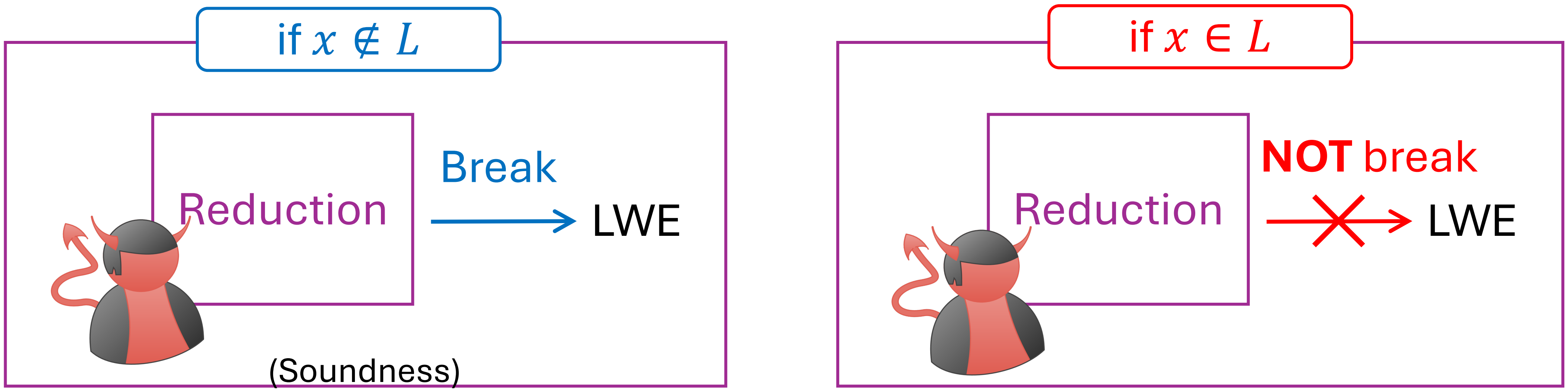

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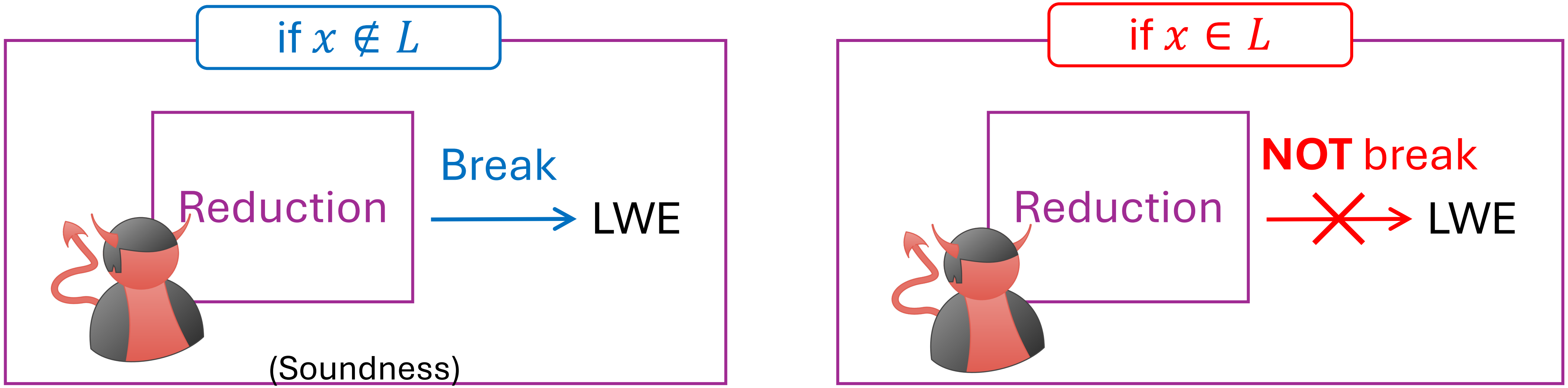

Reduction seems need to "tell" whether $x \notin L$ or $x \in L$.

(Gentry-Wichs: formalize this intuition (with caveat))

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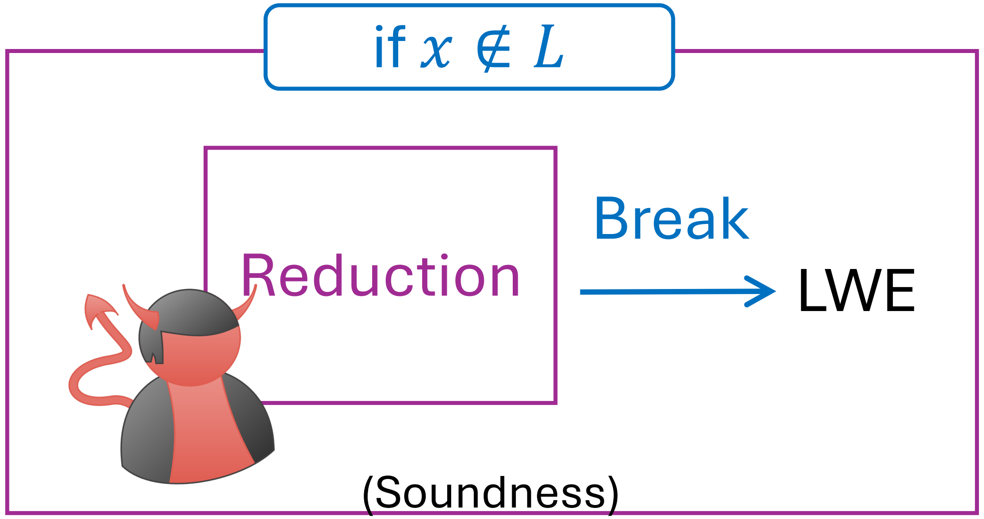

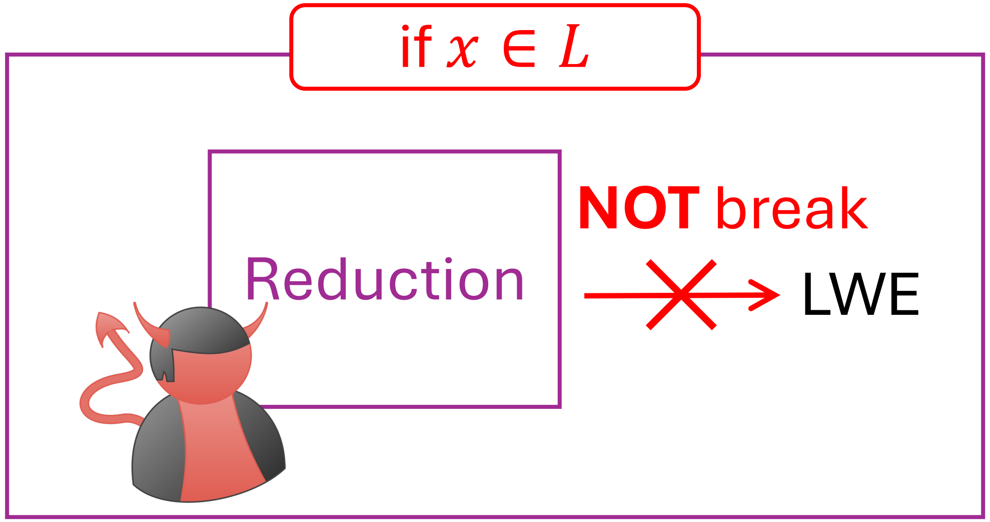

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If the reduction runs in $2^{|w|}$-time $\Rightarrow$ FHE security parameters $\geq |w|$

Not Succinct!

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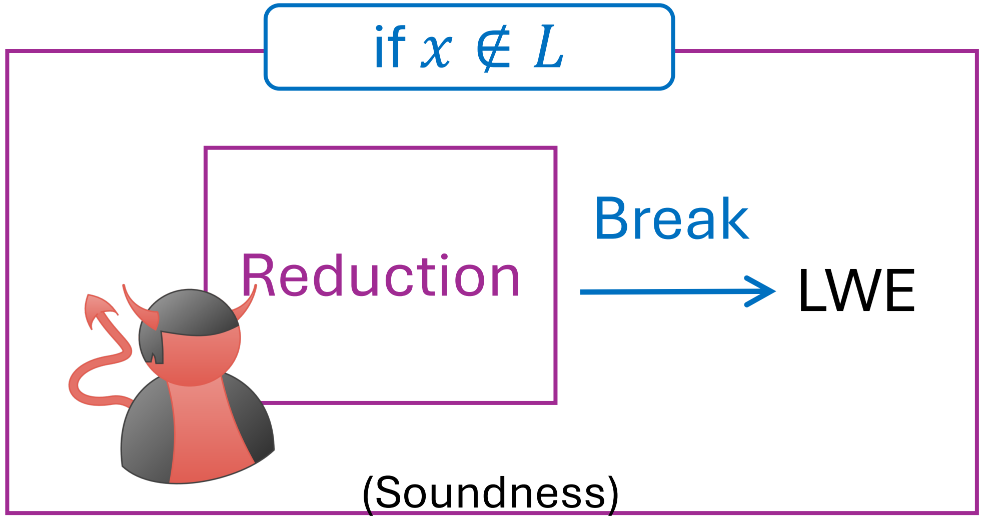

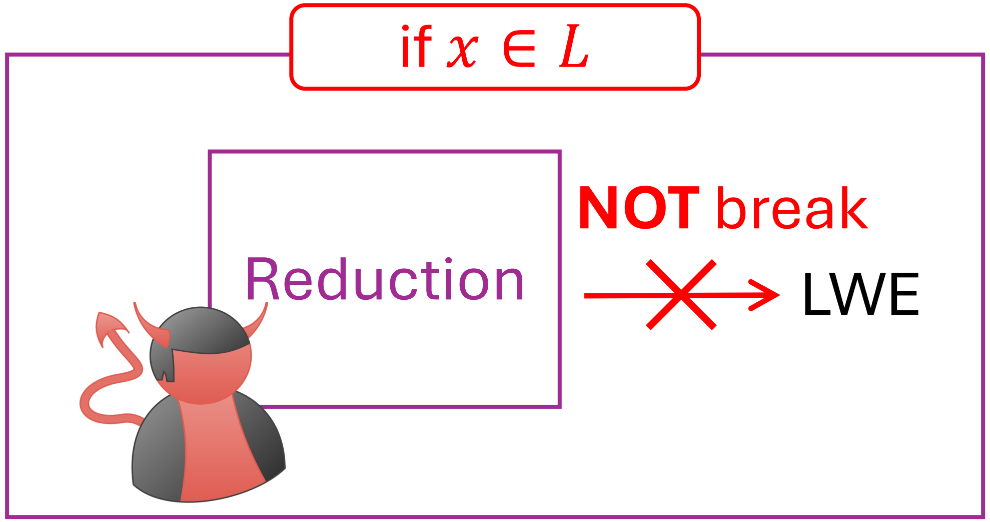

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(poly-time reduction?)

Efficient Reduction via
Logical Structure of the Language [JJ'22]

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Poly-size Extended Frege proof of $R(x,\cdot) = 0$

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⬇

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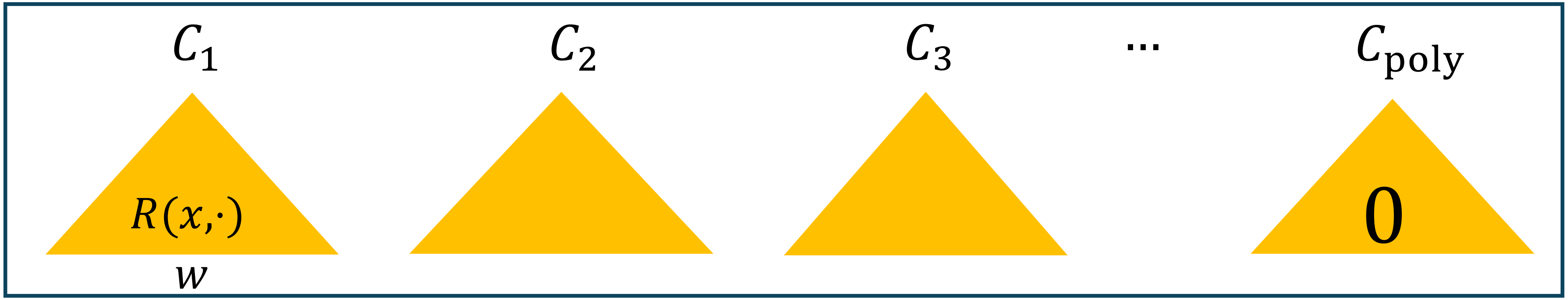

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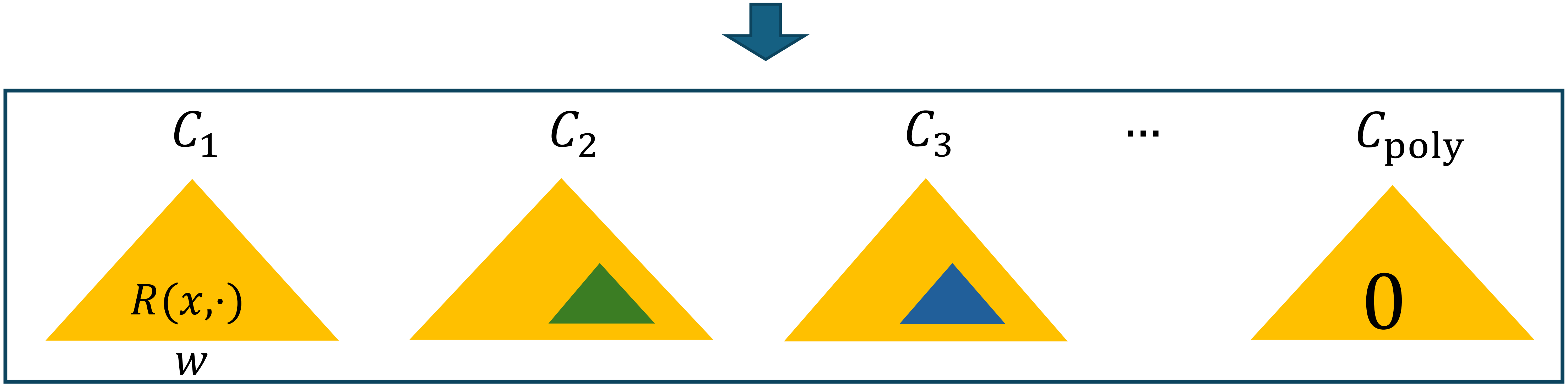

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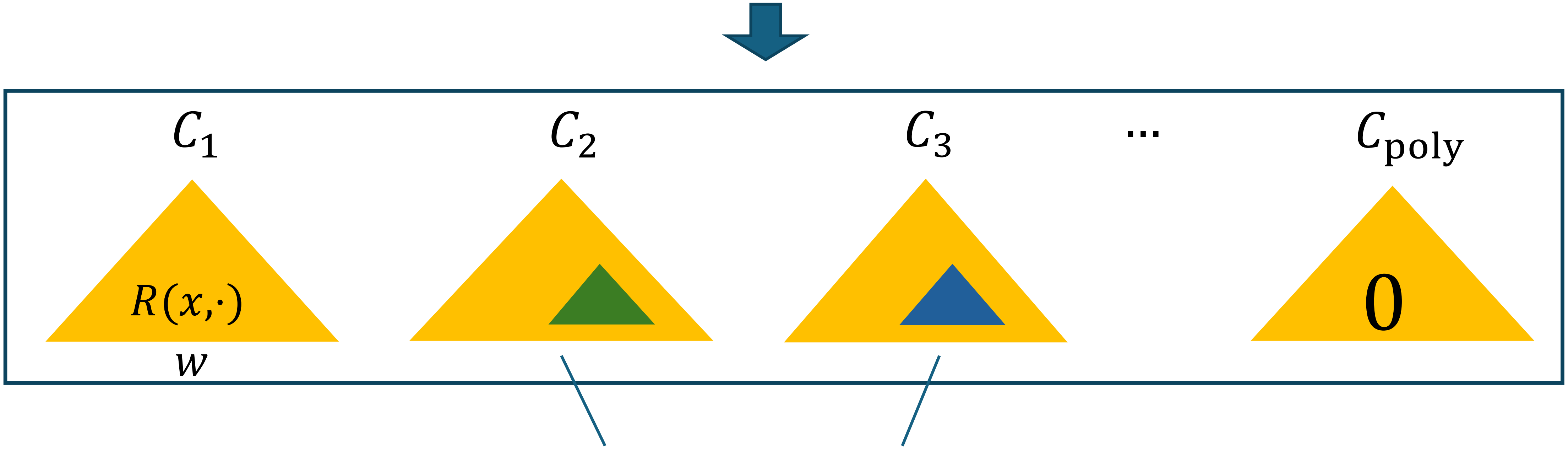
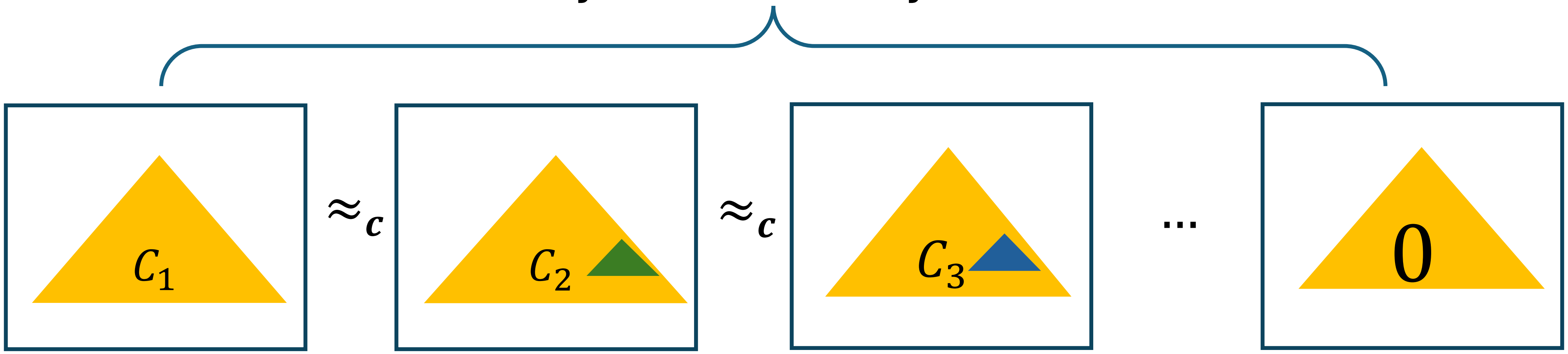


Adjacent pair of circuits are almost the same,
except for a ***O(log n) size*** sub-circuits of the same functionality.

Efficient Reduction via Logical Structure of the Language [JJ'22]

Poly-size Extended Frege proof of $R(x,\cdot) = 0$

Poly. number of hybrids



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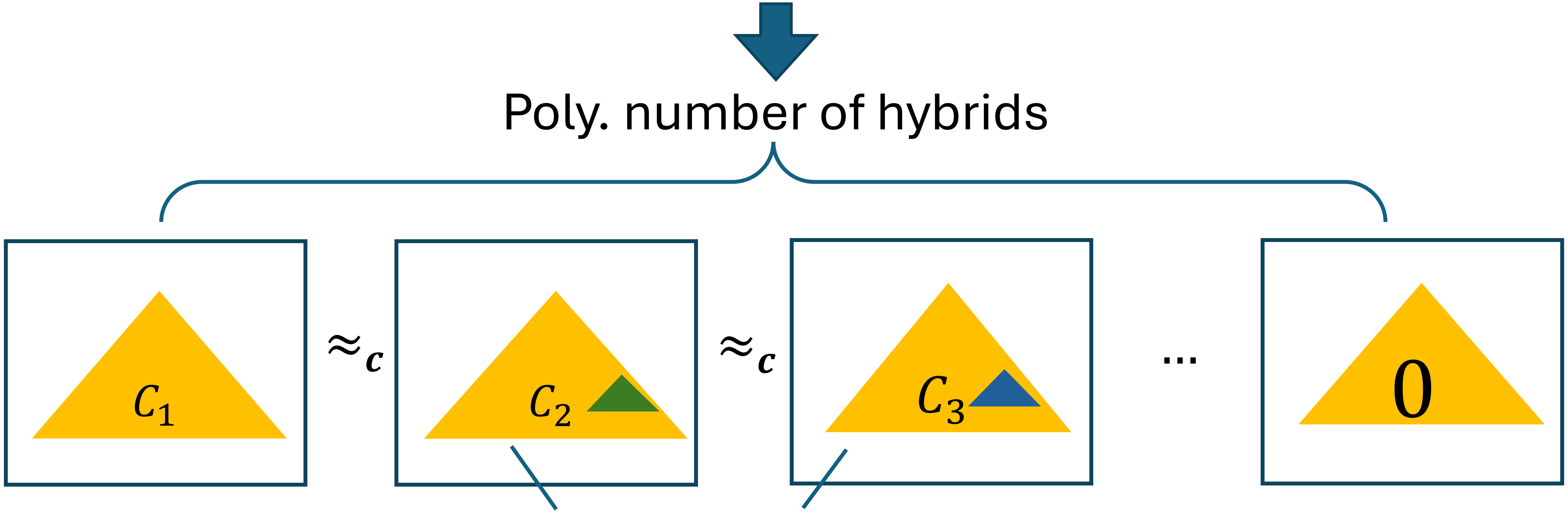

A "Puncturing Argument" with **quasi-polynomial time** reduction

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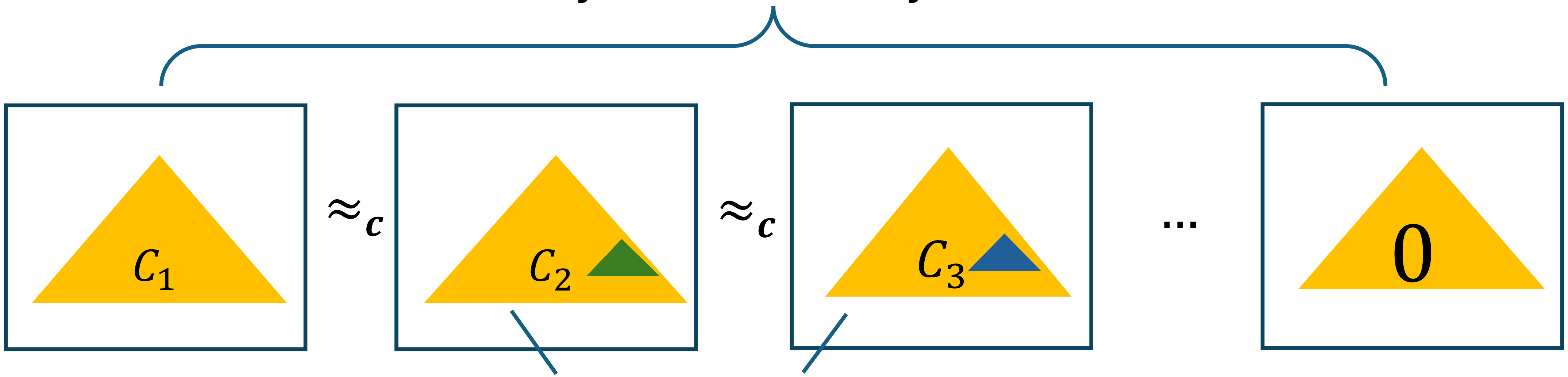

A "Puncturing Argument" with **quasi-polynomial time** reduction

$\Rightarrow$ Total reduction time = **quasi-poly**!

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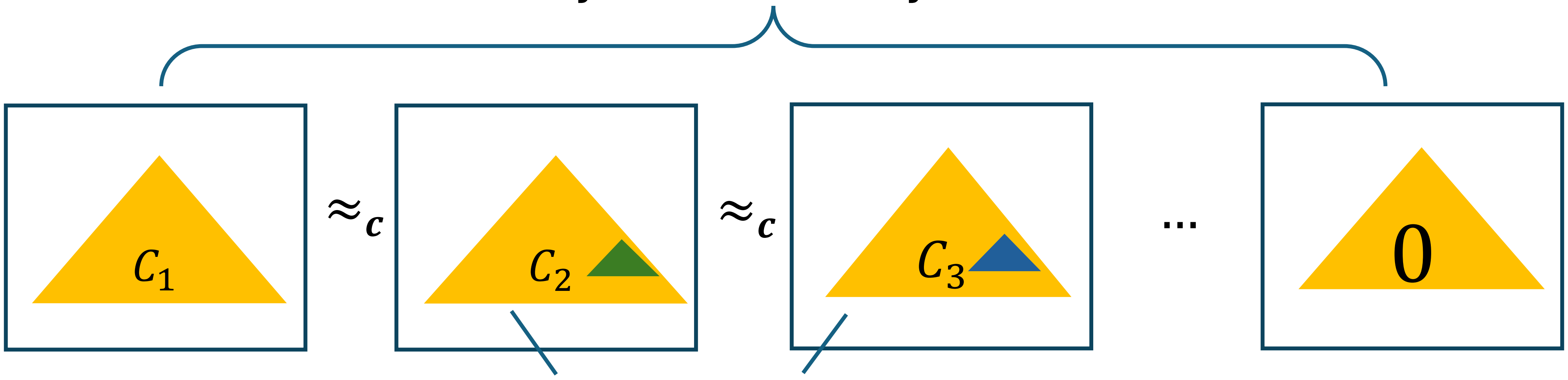


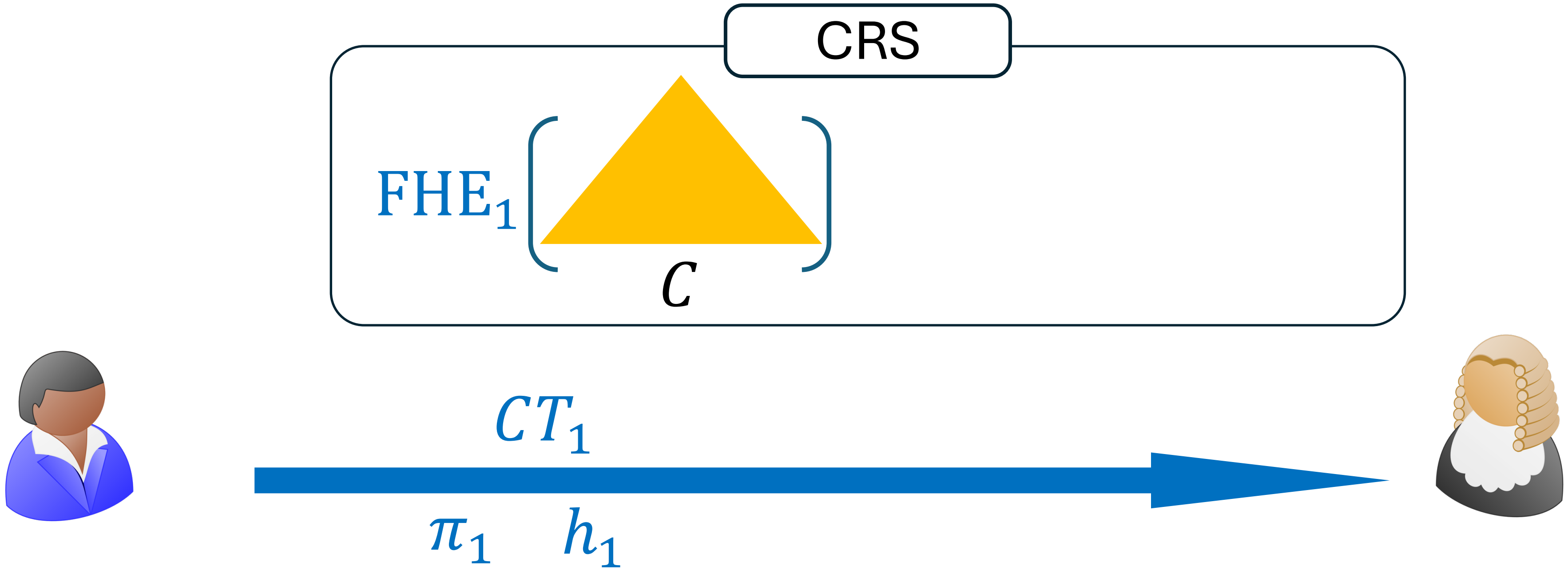
A "Puncturing Argument" with quasi-polynomial time reduction

JJ'22 realizes "puncturing argument" using iO

⇒ Total reduction time = **quasi-poly**!

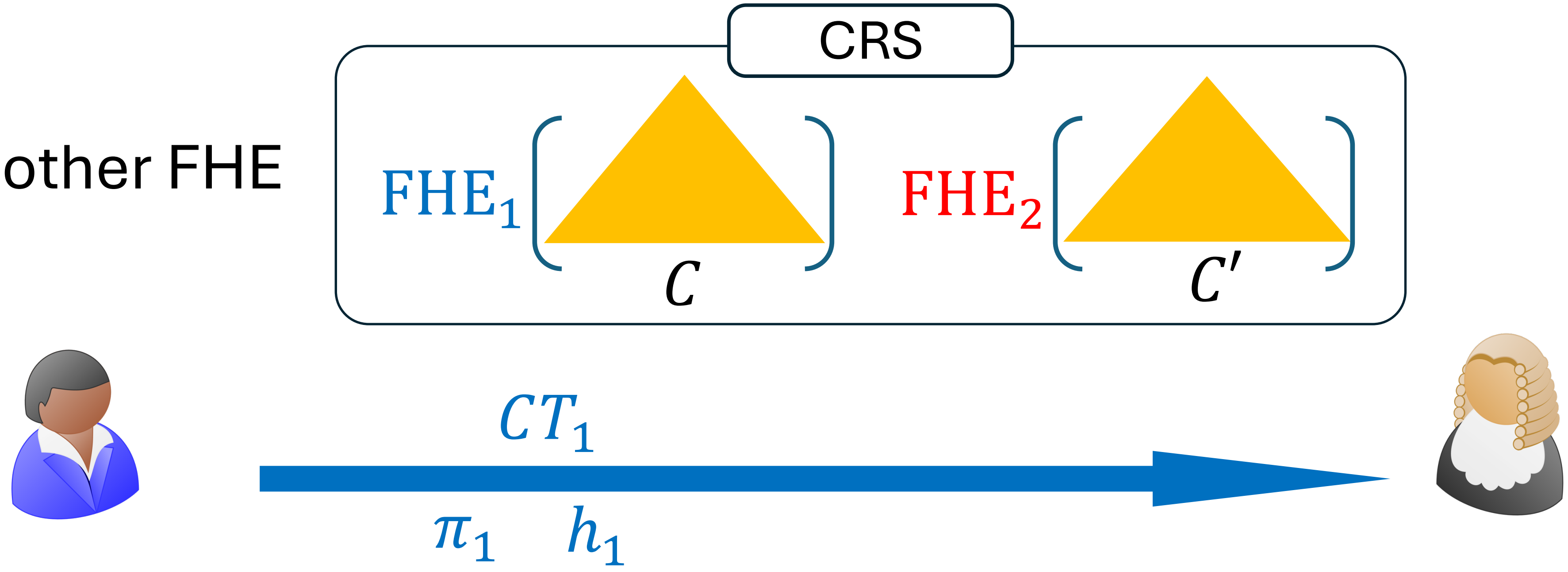
Our Approach

Puncturing Argument, via FHE! (Informal)



Puncturing Argument, via FHE! (Informal)

Add another FHE



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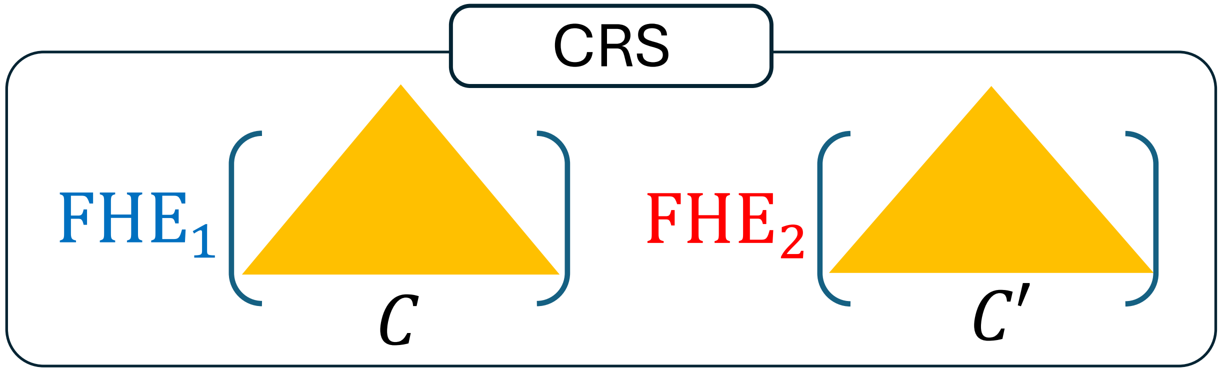

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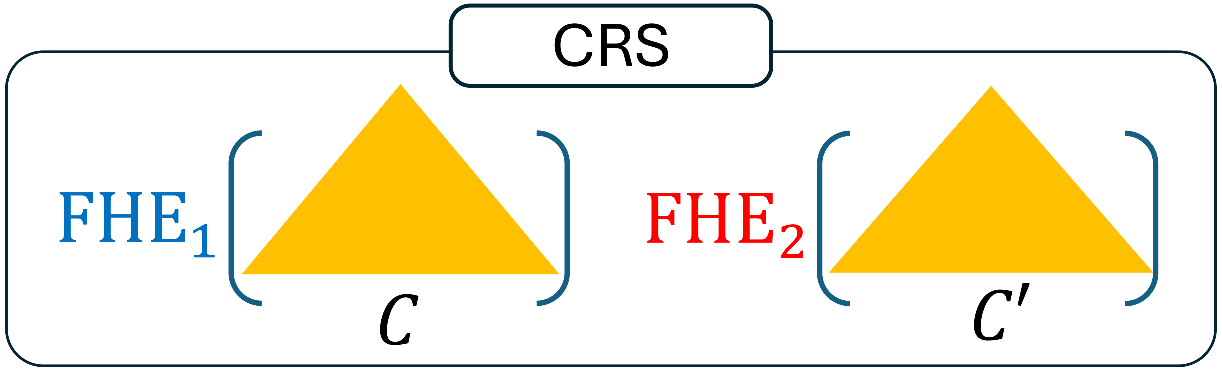

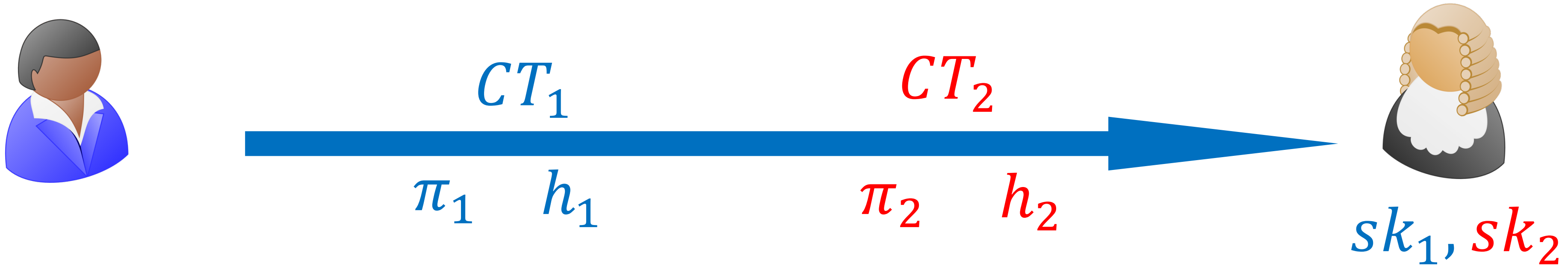

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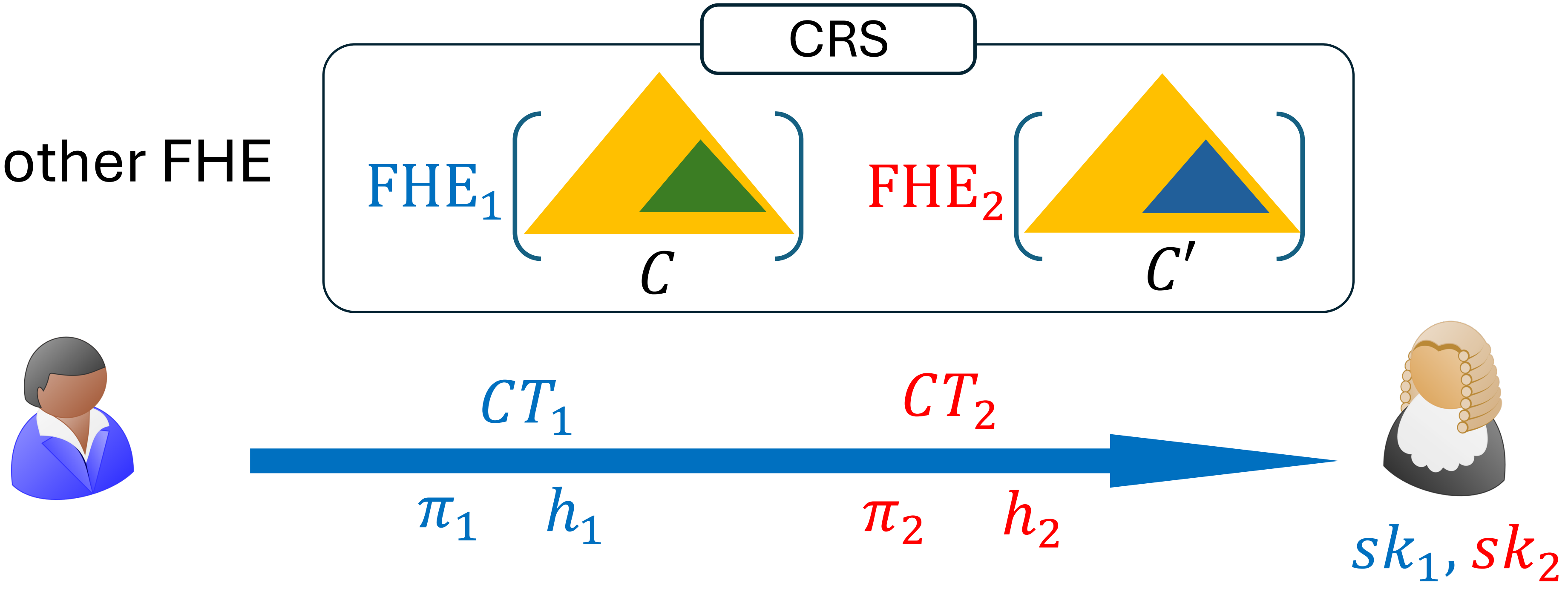

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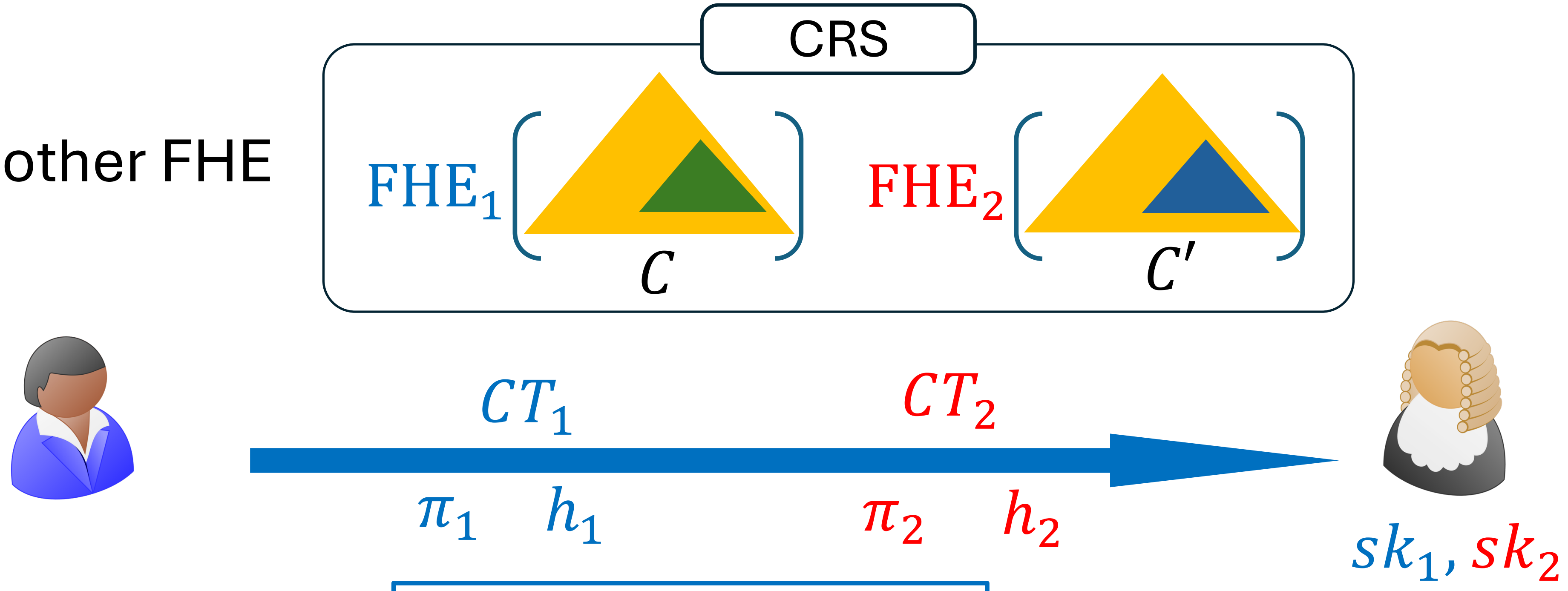


Puncturing Argument

If C and C' are almost the same except for a functionality equivalent ***O(log n)*** sub-ckt, then $\mathrm{Dec}(sk_1, CT_1) = 1$ implies $\mathrm{Dec}(sk_2, CT_2) = 1$, using **poly-secure BARG.**

Proving Soundness via Puncturing Argument

...

Proving Soundness via Puncturing Argument

Poly-size Extended Frege proof of $R(x, w) = 0$

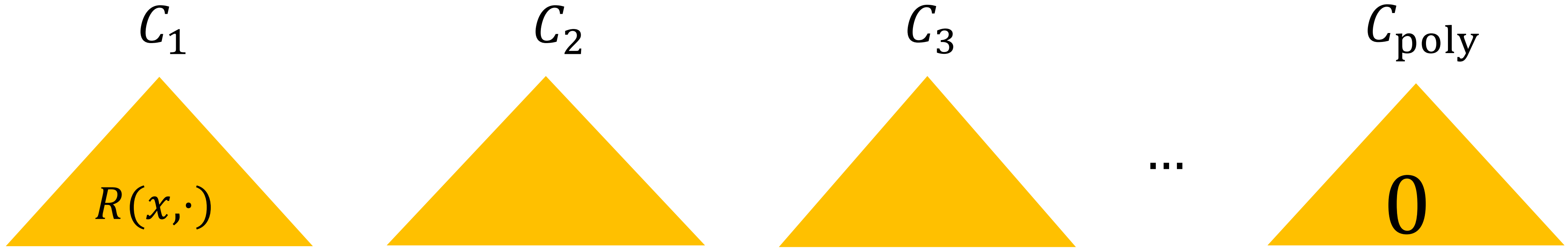
$\Downarrow$ [JJ22]

...

Proving Soundness via Puncturing Argument

Poly-size Extended Frege proof of $R(x, w) = 0$

[JJ22]



Proving Soundness via Puncturing Argument

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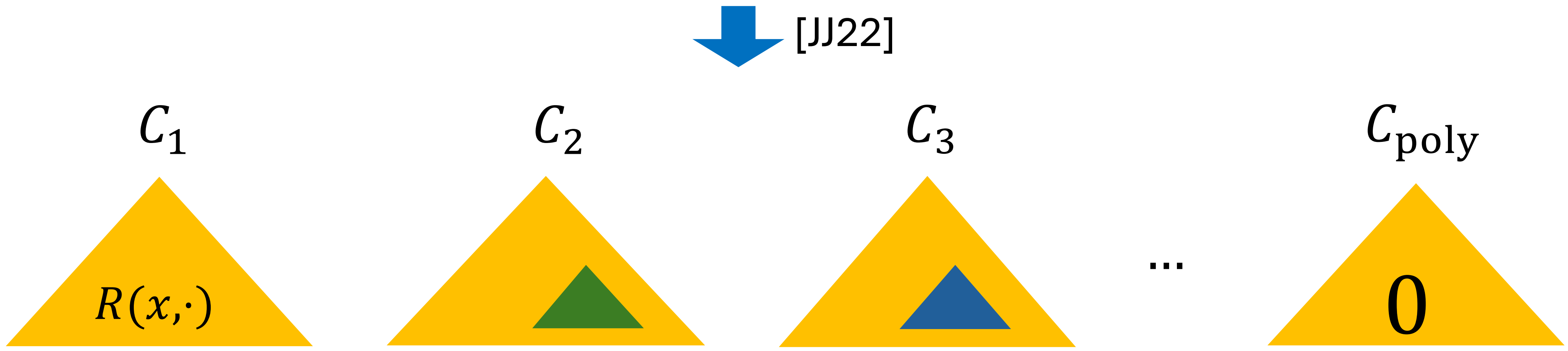

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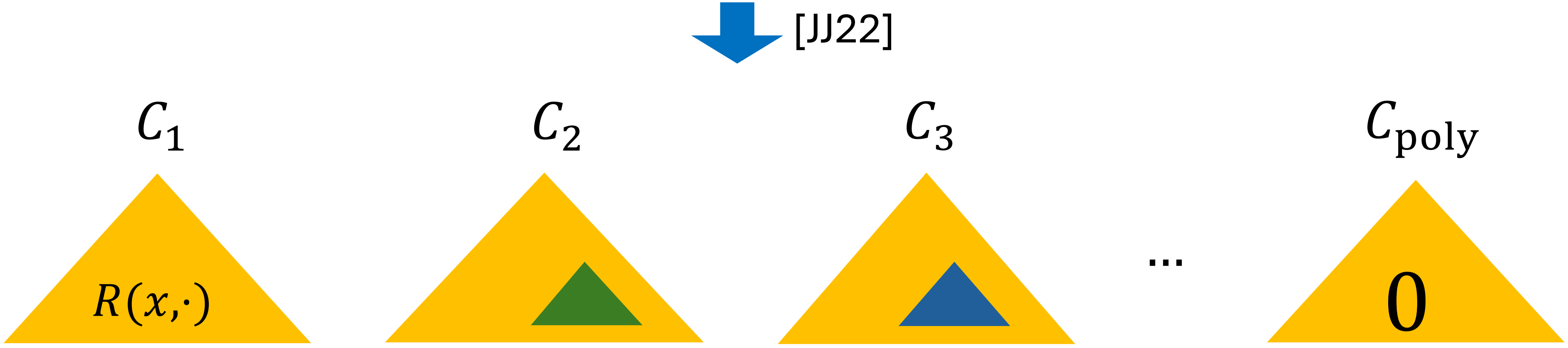
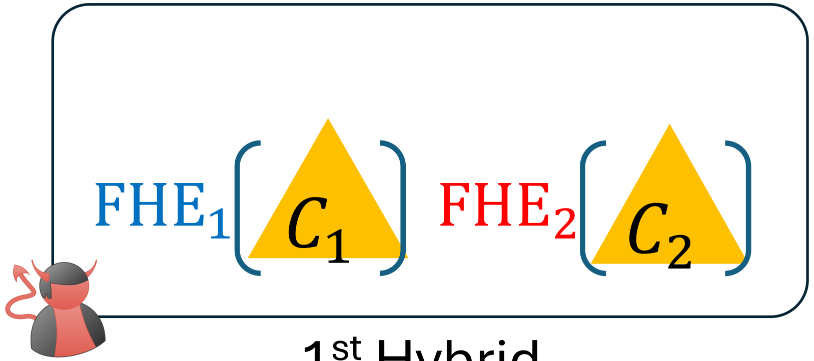
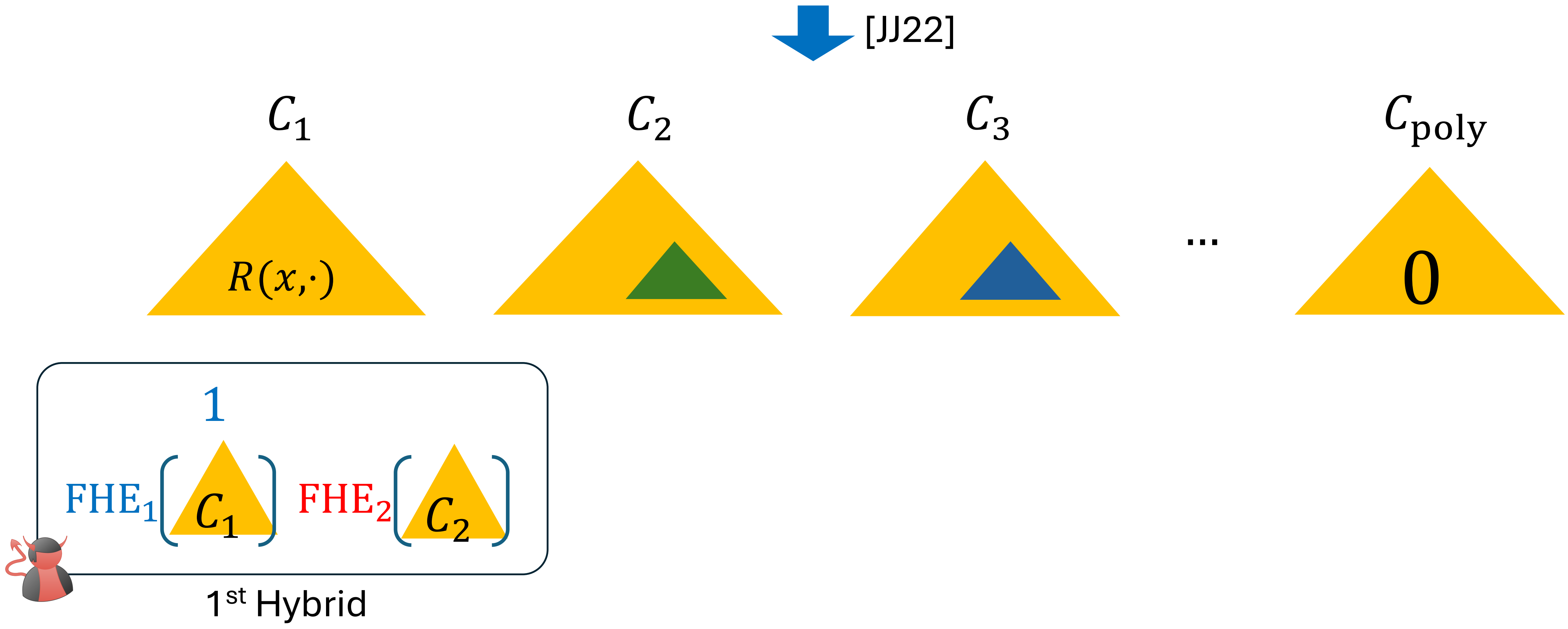




1st Hybrid

Proving Soundness via Puncturing Argument

Poly-size Extended Frege proof of $R(x,w) = 0$



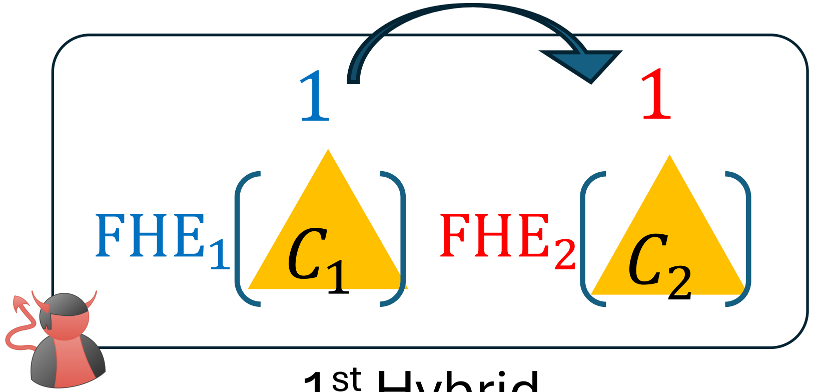
Proving Soundness via Puncturing Argument

Poly-size Extended Frege proof of $R(x, w) = 0$

[JJ22]

...

Puncturing Argument



1^{st} Hybrid

Proving Soundness via Puncturing Argument

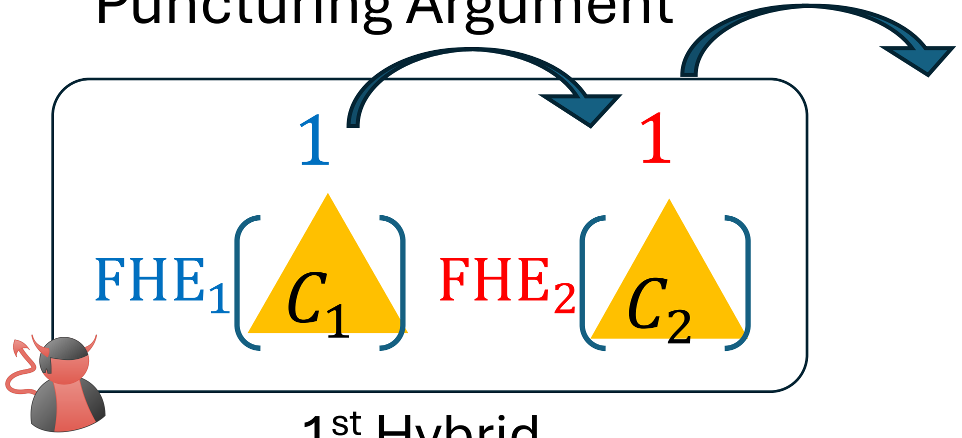
Poly-size Extended Frege proof of $R(x, w) = 0$

[JJ22]

...

FHE_1 semantic security

Puncturing Argument

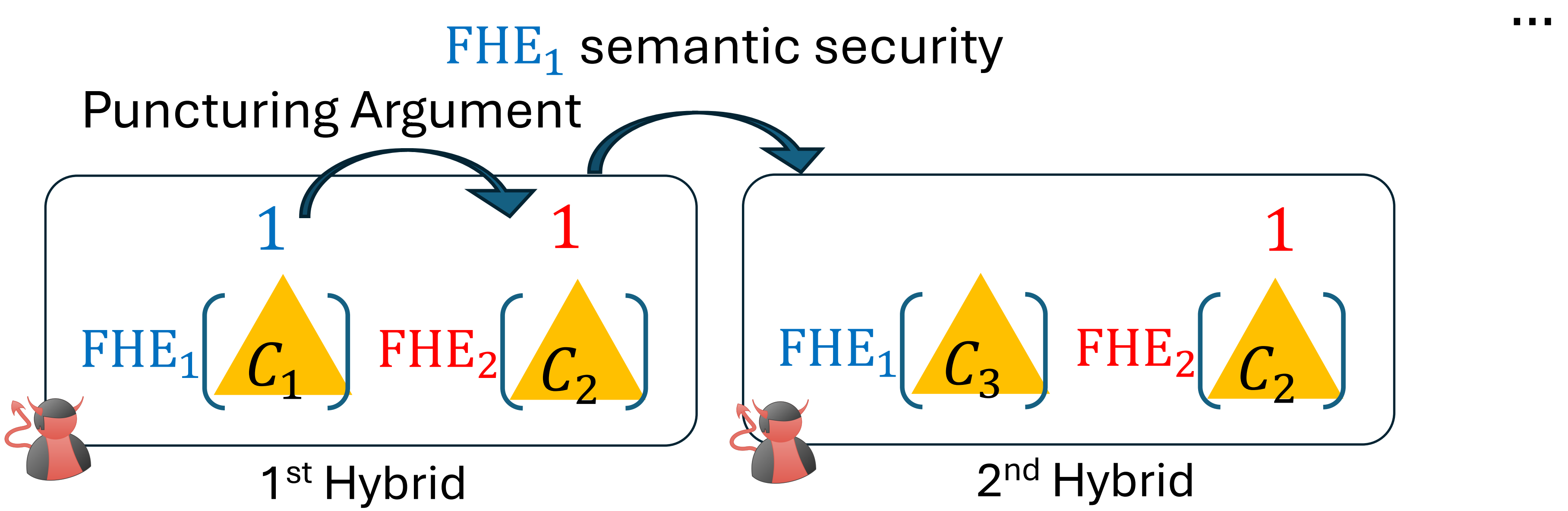


1st Hybrid

Proving Soundness via Puncturing Argument

Poly-size Extended Frege proof of $R(x, w) = 0$

$\Downarrow$ [JJ22]



Proving Soundness via Puncturing Argument

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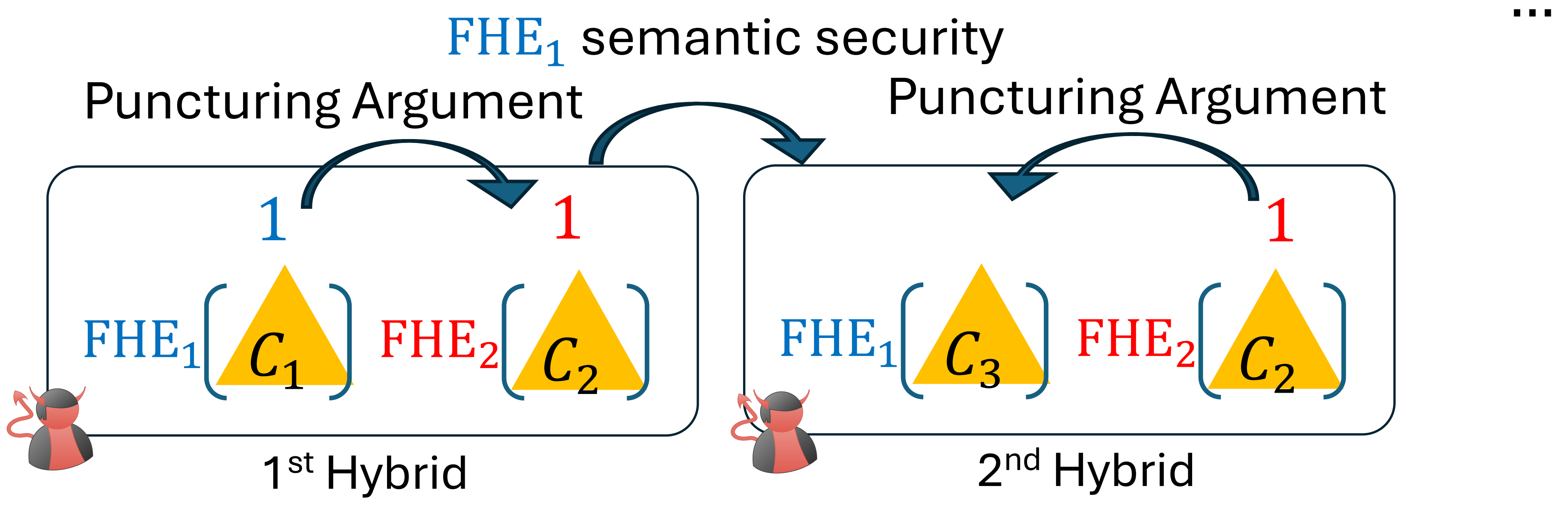
[JJ22]



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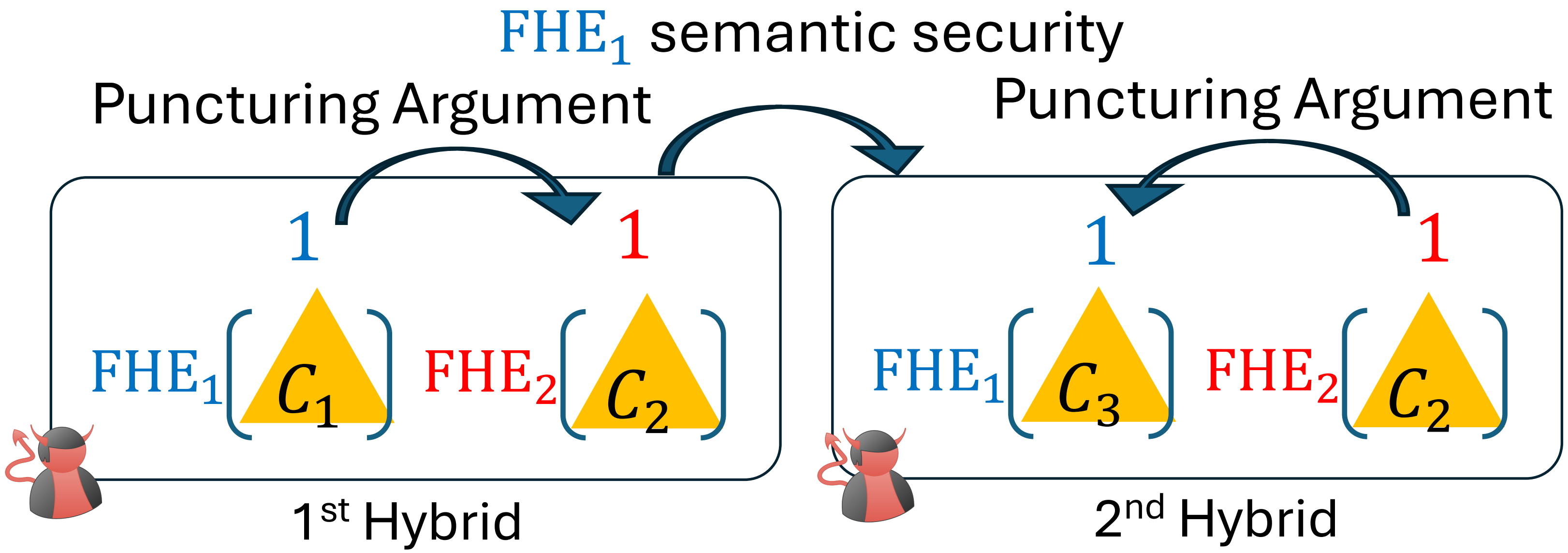
[JJ22]

...



Proving Soundness via Puncturing Argument

Poly-size Extended Frege proof of $R(x, w) = 0$

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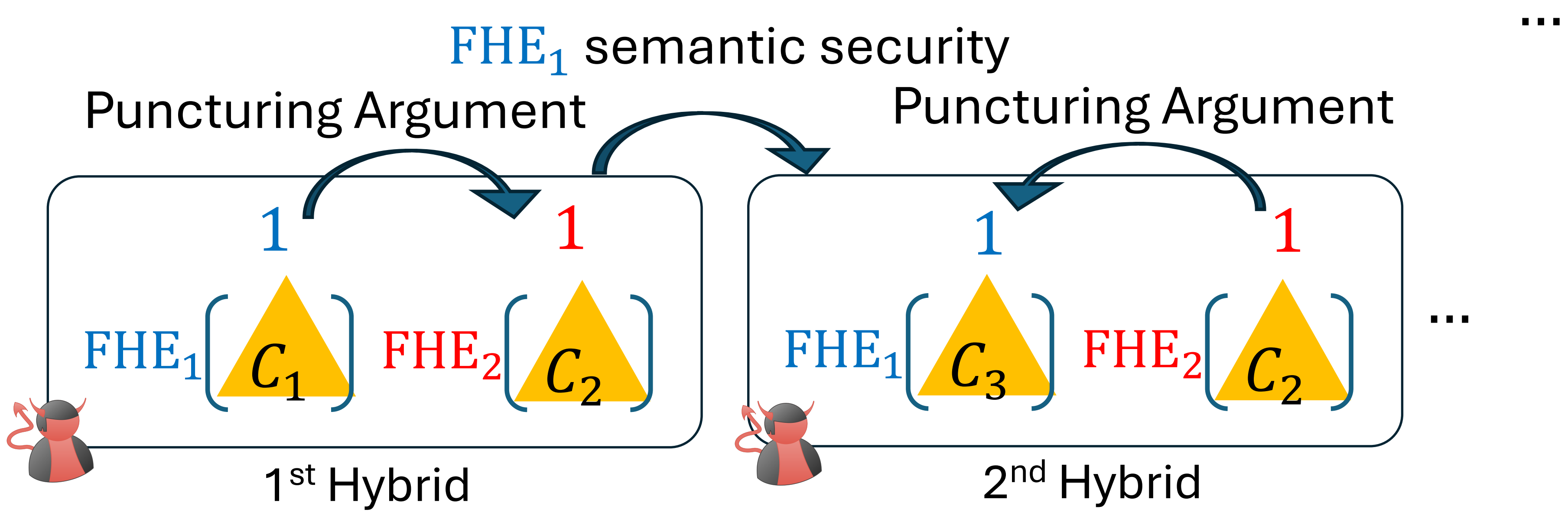

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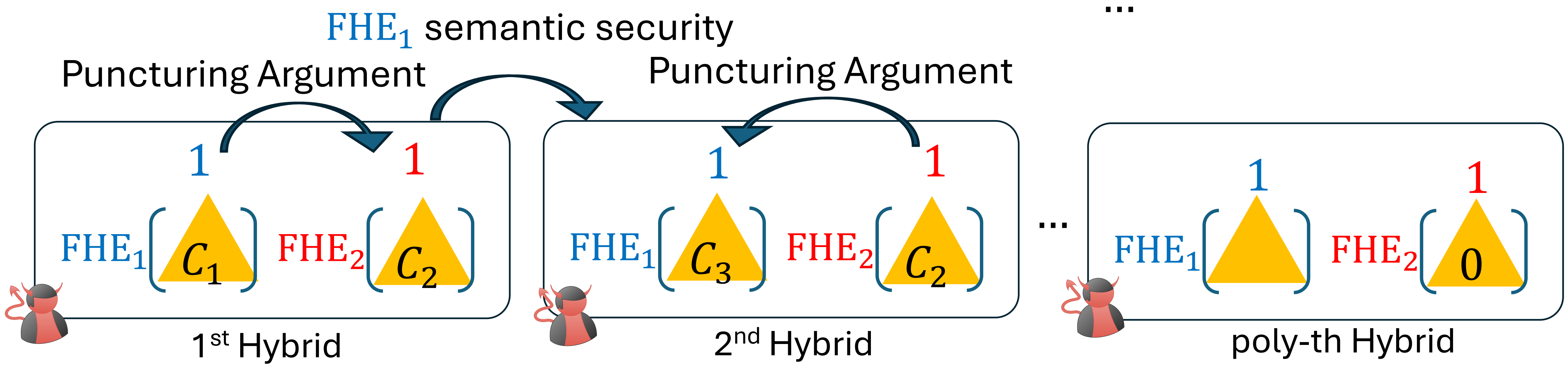

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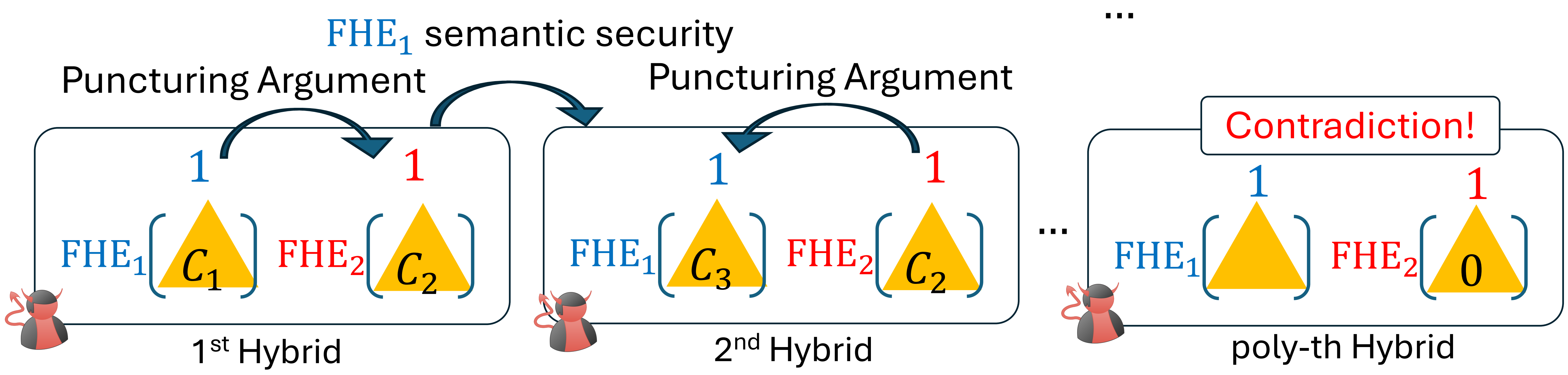

Rest of the Talk

- Proof of Puncturing Argument
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Recall: Somewhere Statistical Binding (SSB) Hash

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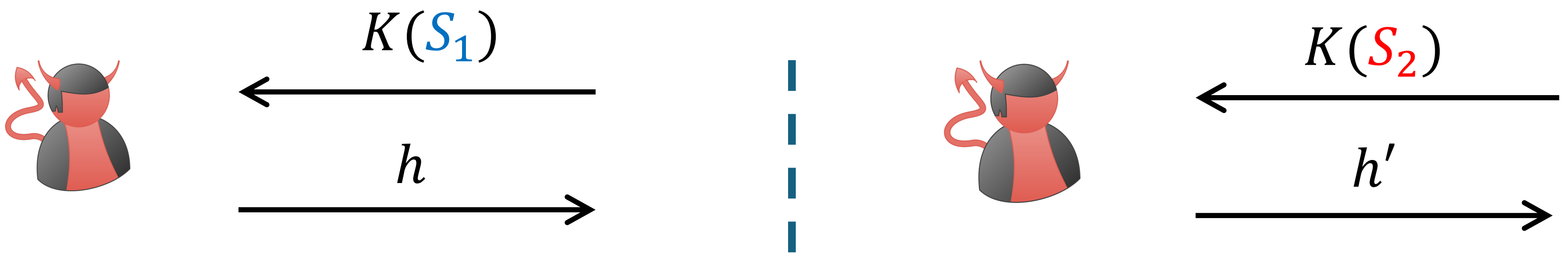

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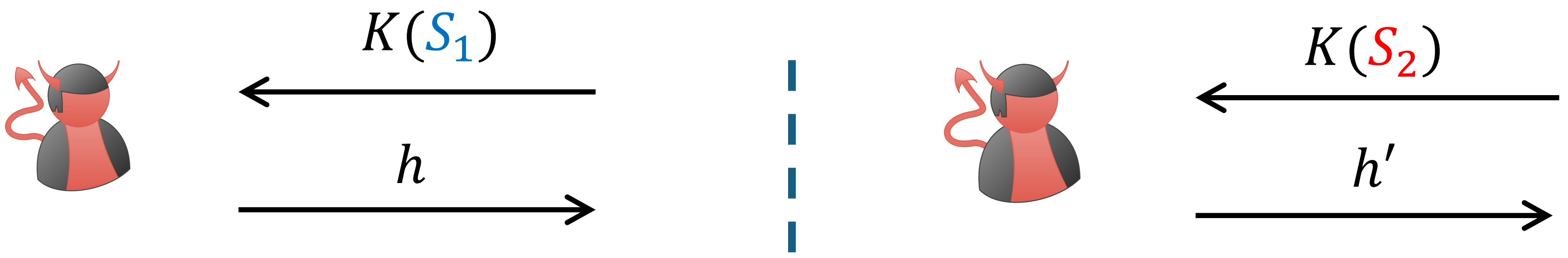
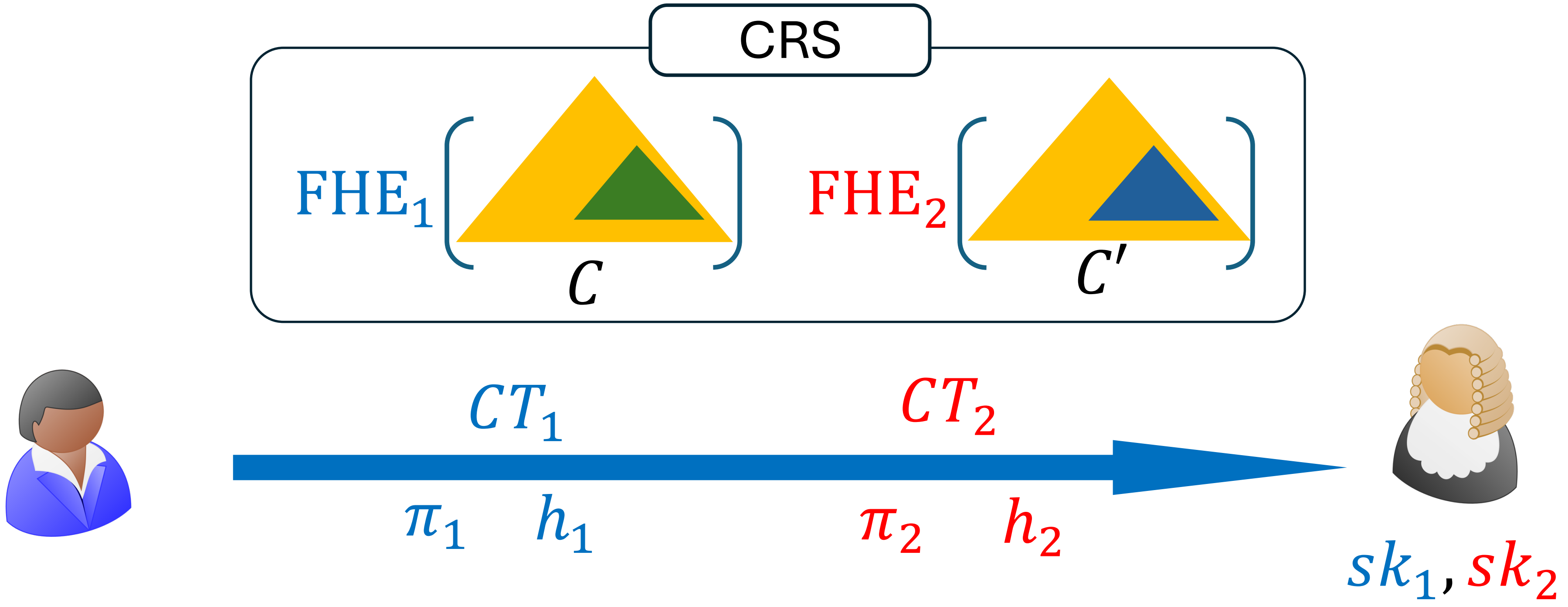


$$m_{S_1 \cap S_2} \approx_c m'_{S_1 \cap S_2}$$

Recall: Puncturing Argument



If C and C' are almost the same except for a functionality equivalent ***O(log n)*** sub-ckt, then $\mathrm{Dec}(sk_1, CT_1) = 1$ implies $\mathrm{Dec}(sk_2, CT_2) = 1$, assuming **poly-secure BARG.**

A Simplified View via Local Assignment Generator

[BHK17]

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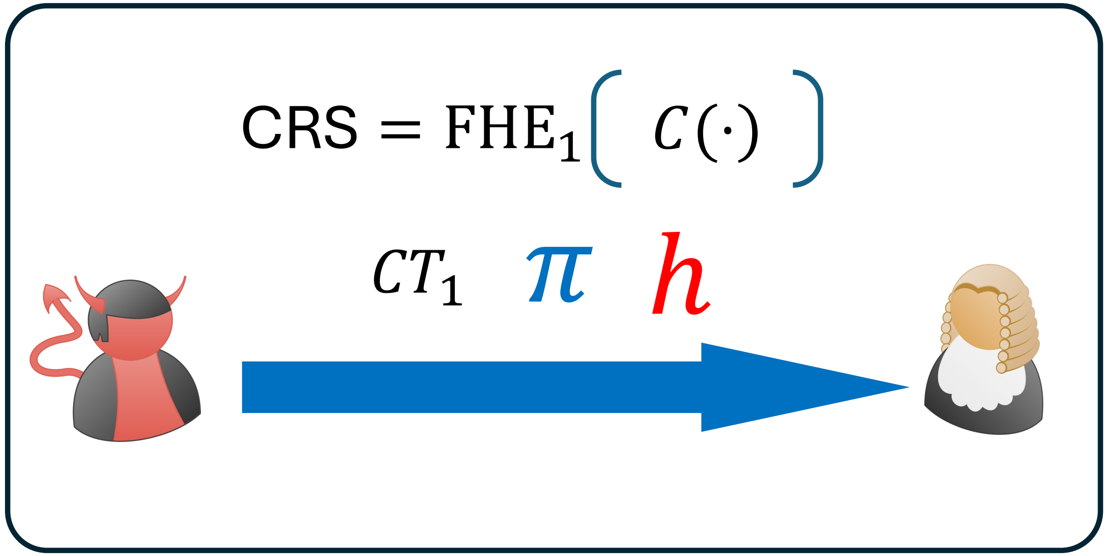

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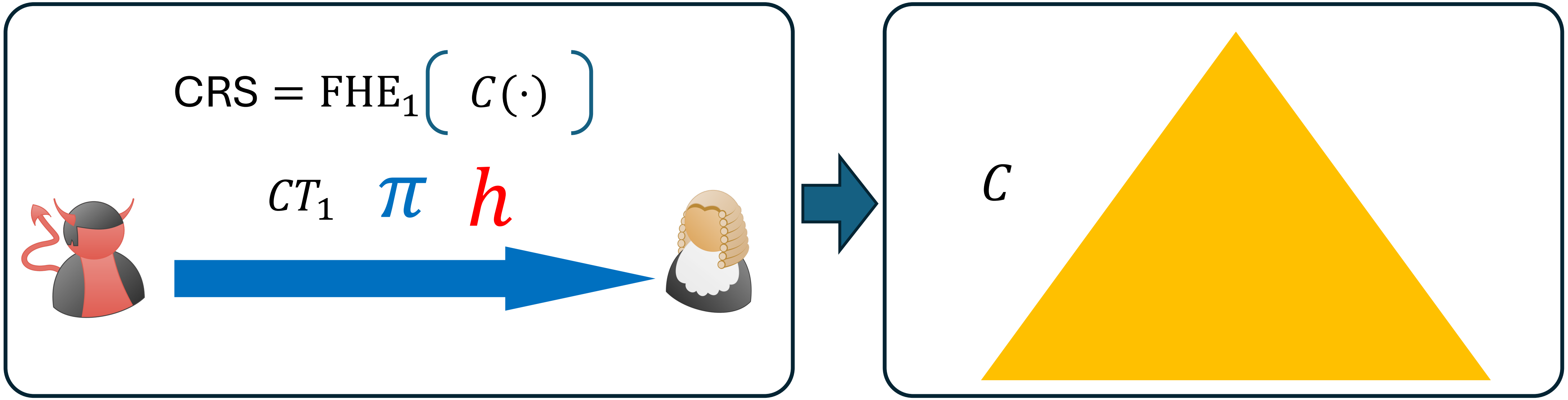

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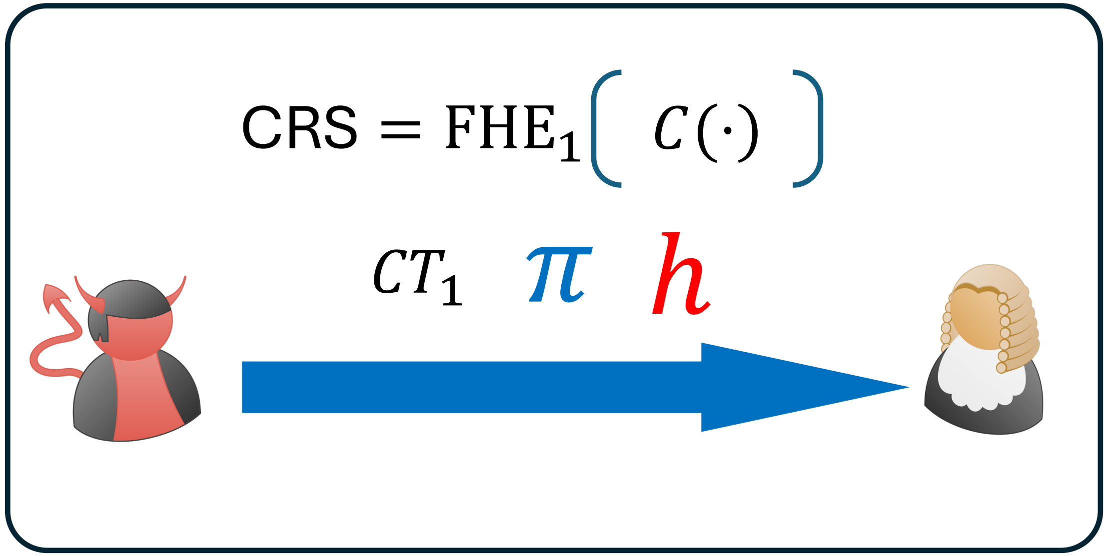

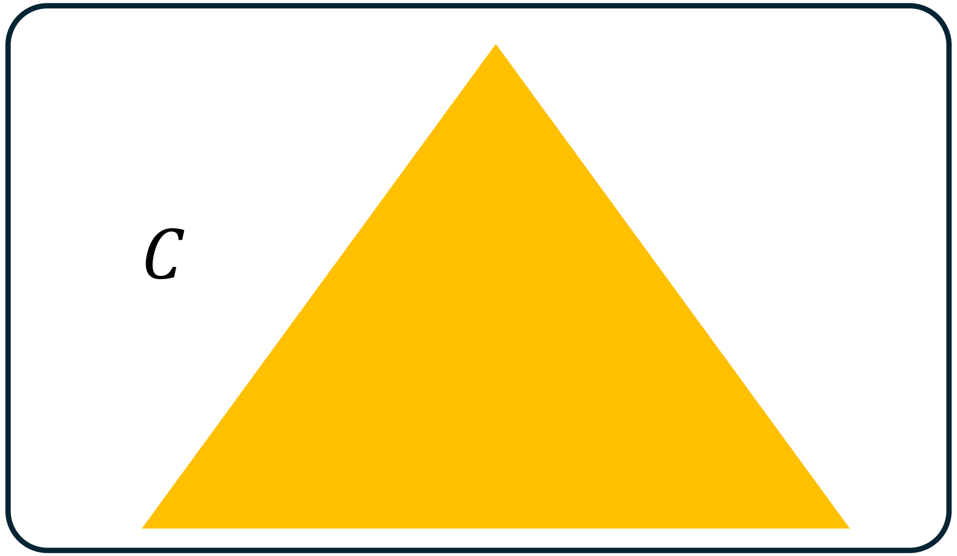

Local Assignment Generator: $\mathrm{Gen}(S) := \mathrm{FHE}_1.\mathrm{Dec}\big(sk, \mathrm{Ext}(\mathrm{td}, h)\big)$

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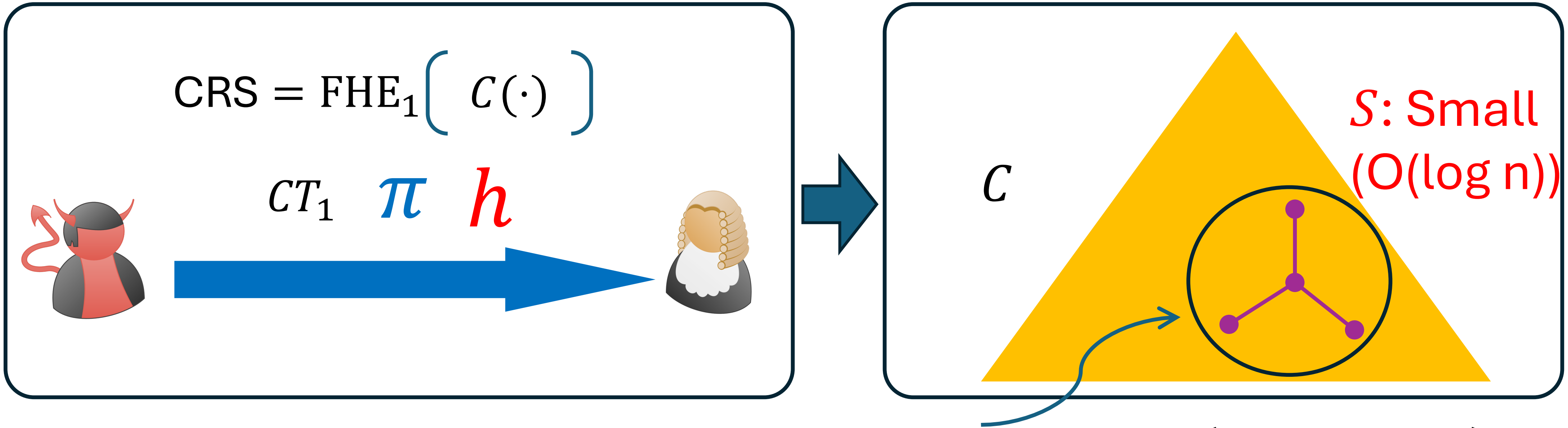

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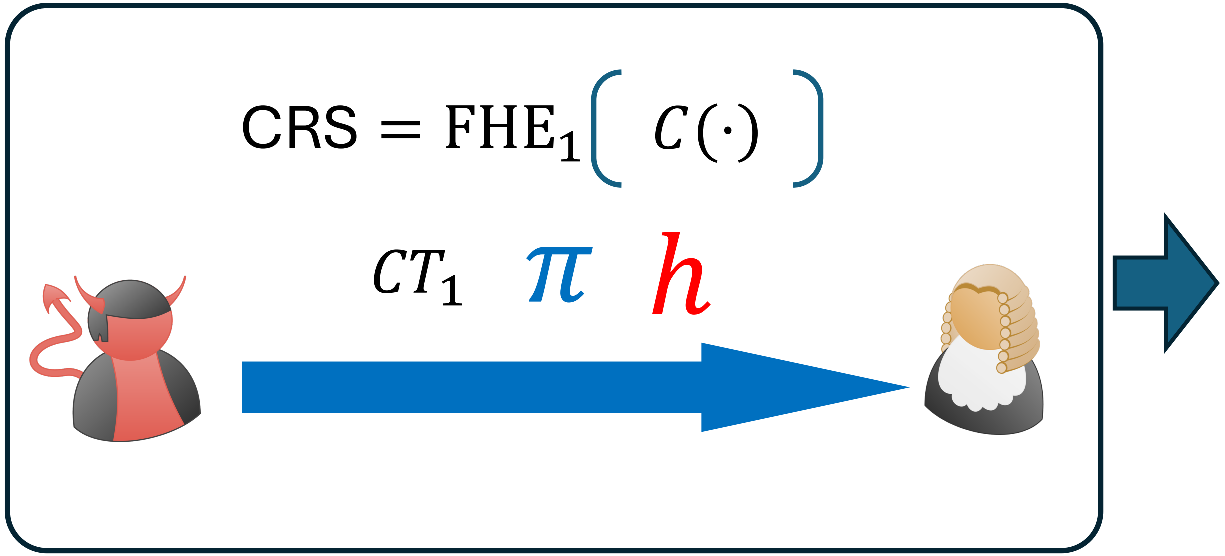
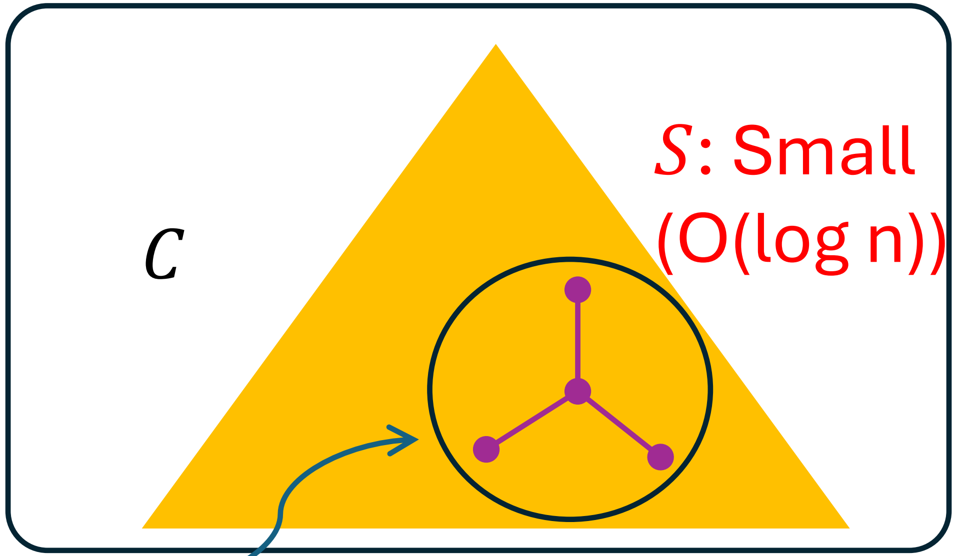


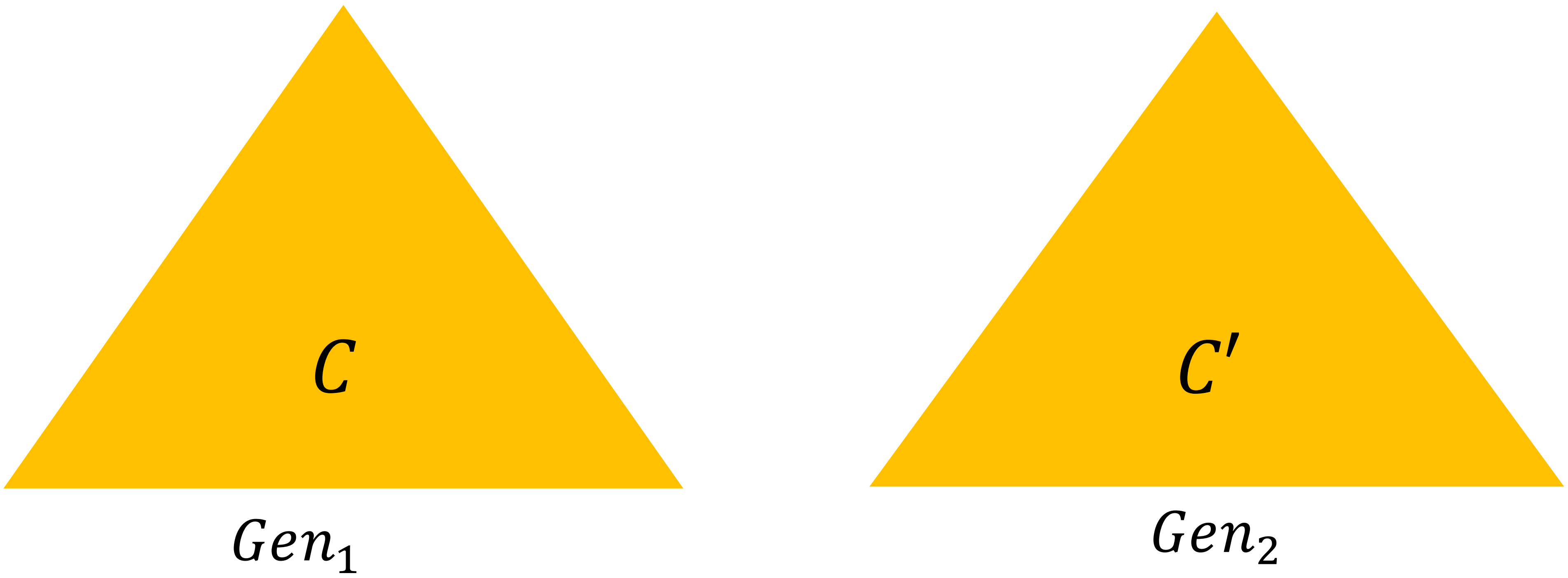


Local Assignment Generator: $\text{Gen}(S) \coloneqq \text{FHE}_1.\text{Dec}\big(sk, \text{Ext}(\text{td}, h)\big)$

Properties of Gen

- No-Signaling (from No-Signaling property of SSB)
- Extracted wire values satisfy the gates in S (via Soundness of BARGs)

Puncturing Argument, Rephrased



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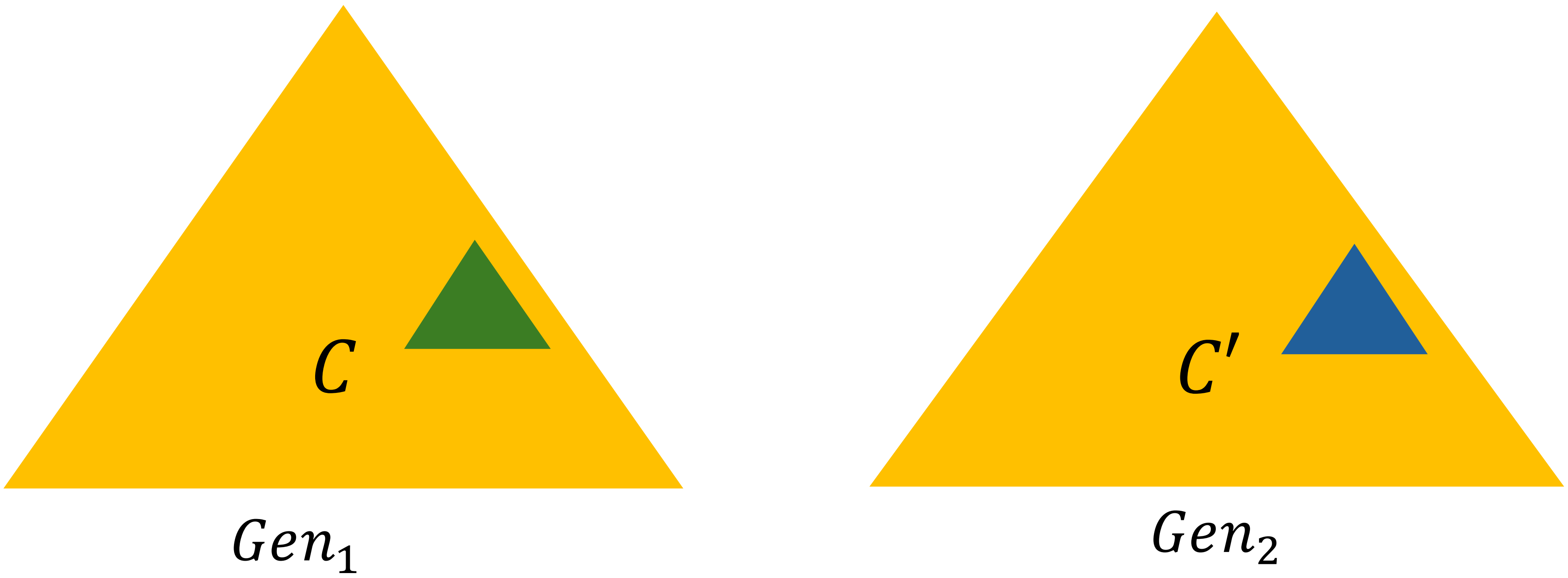
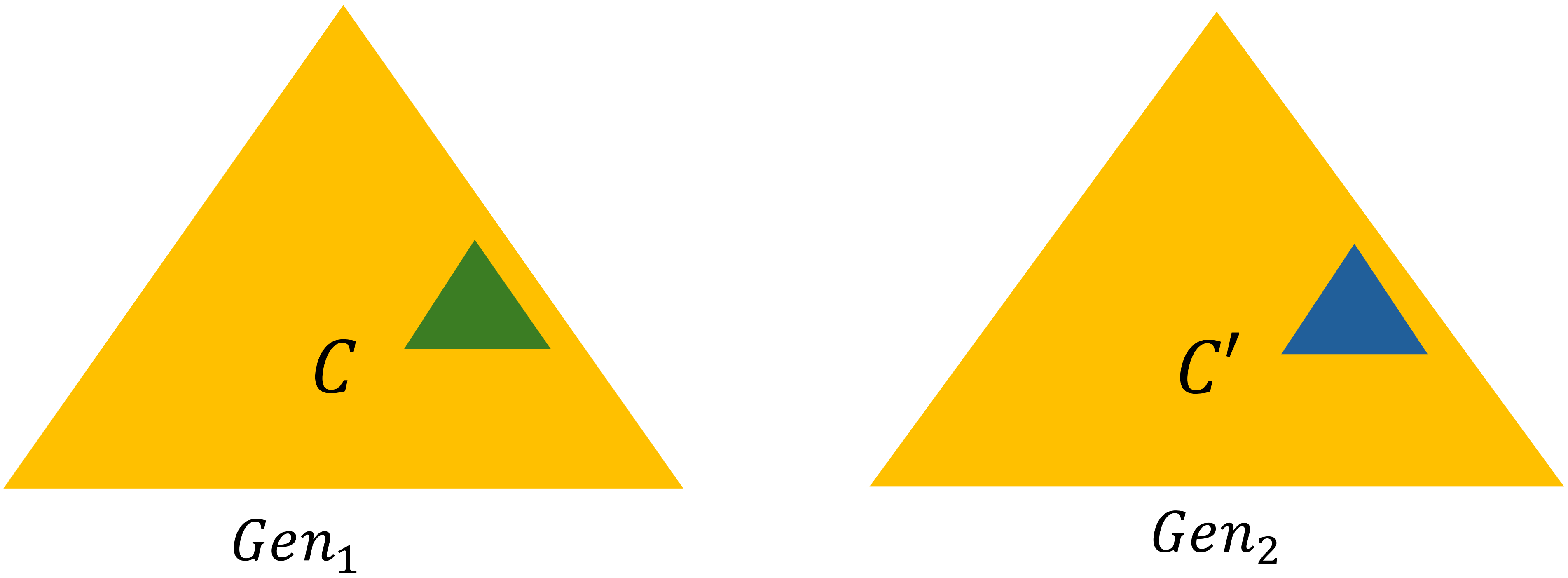


Puncturing Argument, Rephrased

Extracted output wire = 1



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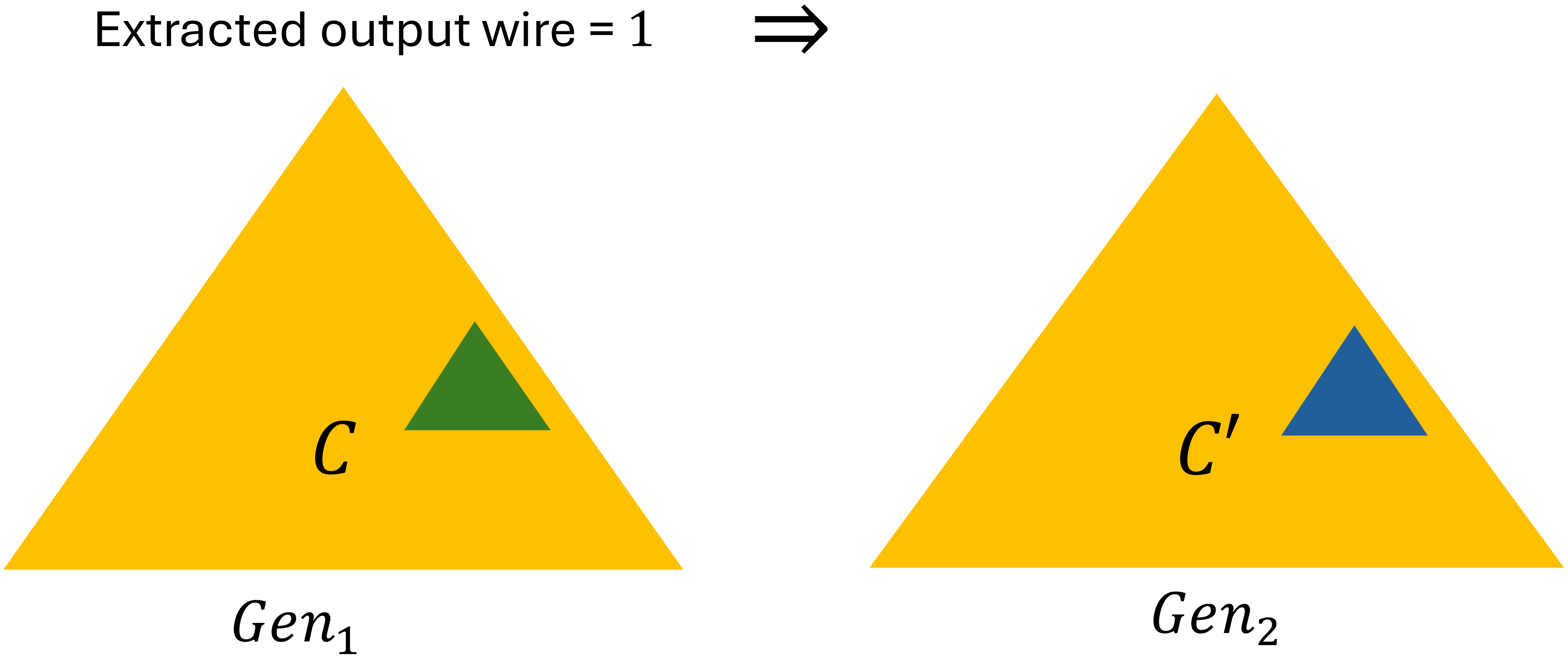

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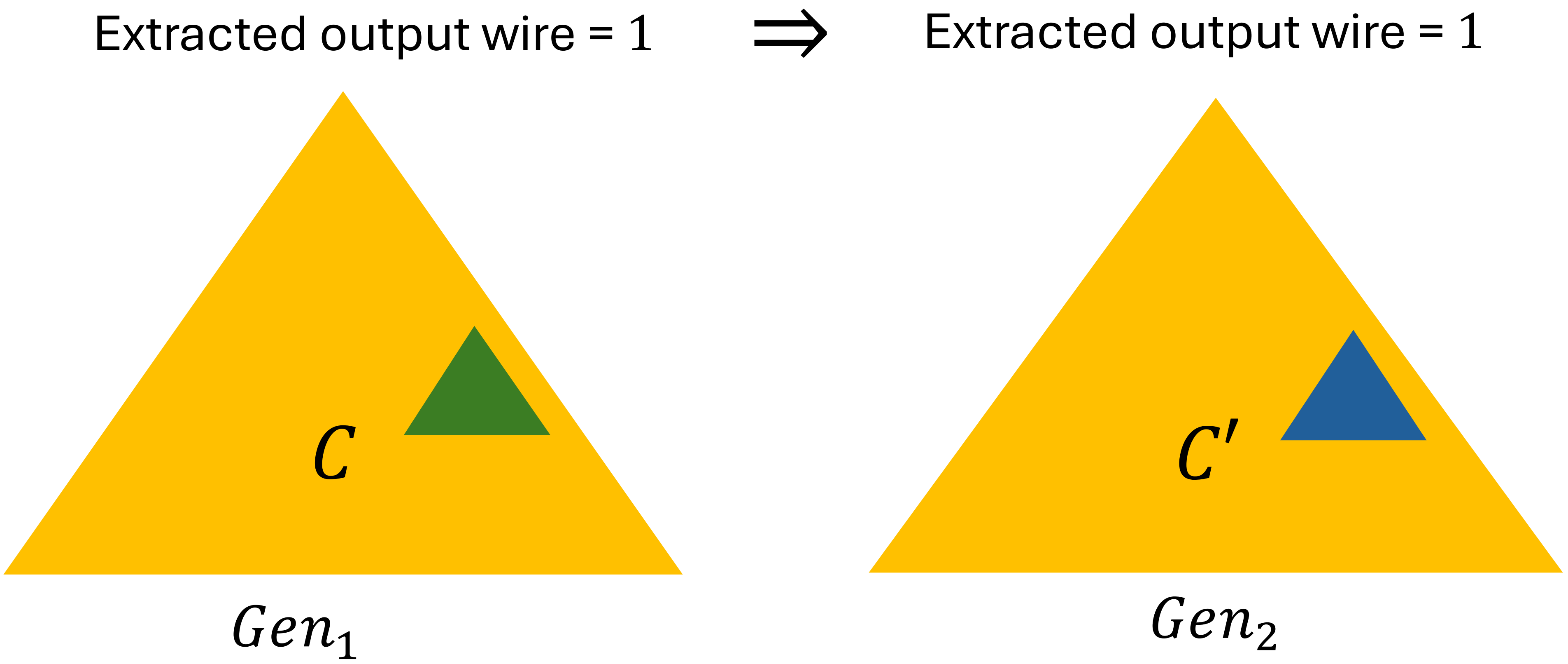
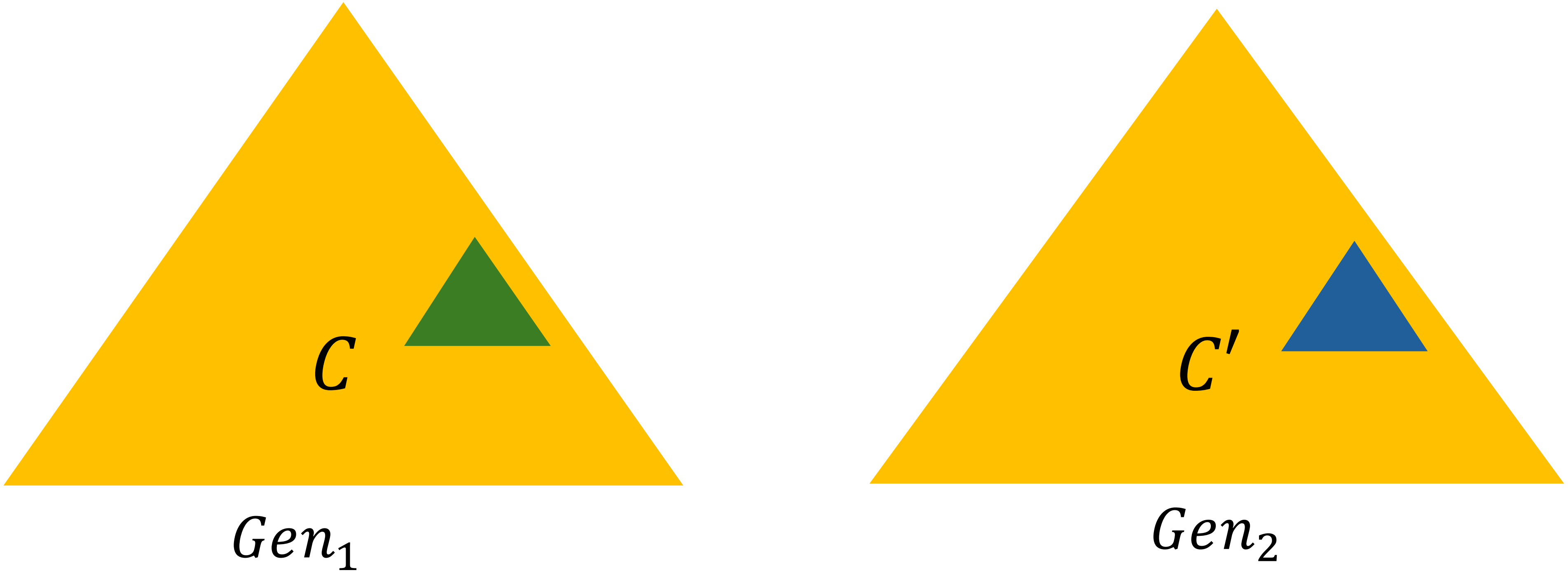


Puncturing Argument, Rephrased

We want to prove

Extracted output wire = 1 $\Rightarrow$ Extracted output wire = 1



Blueprint of the Proof

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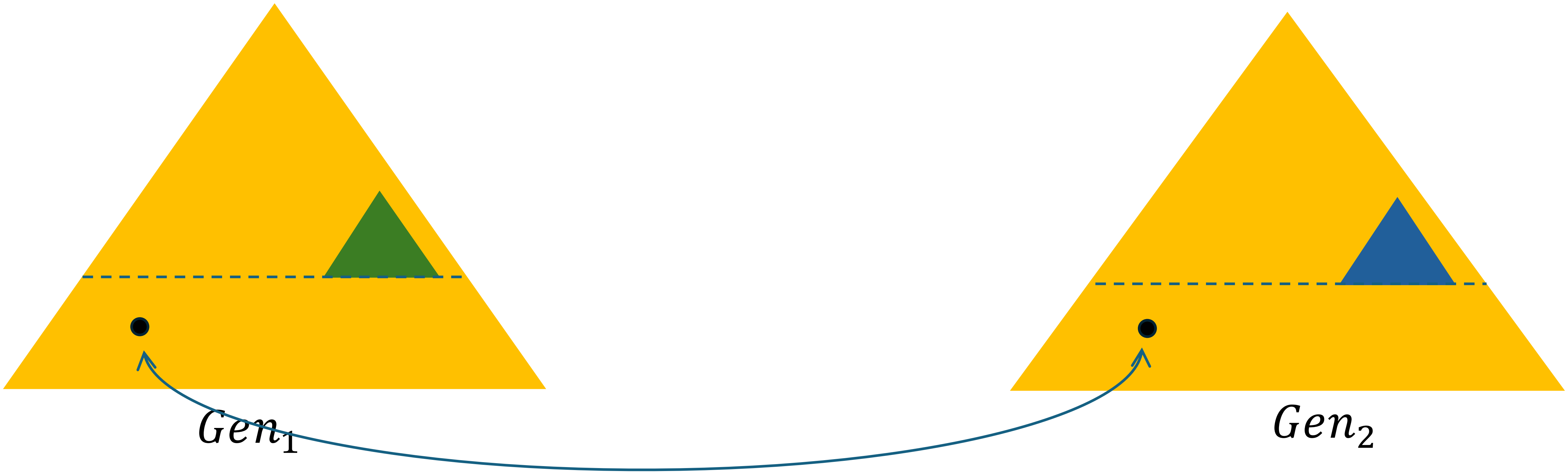
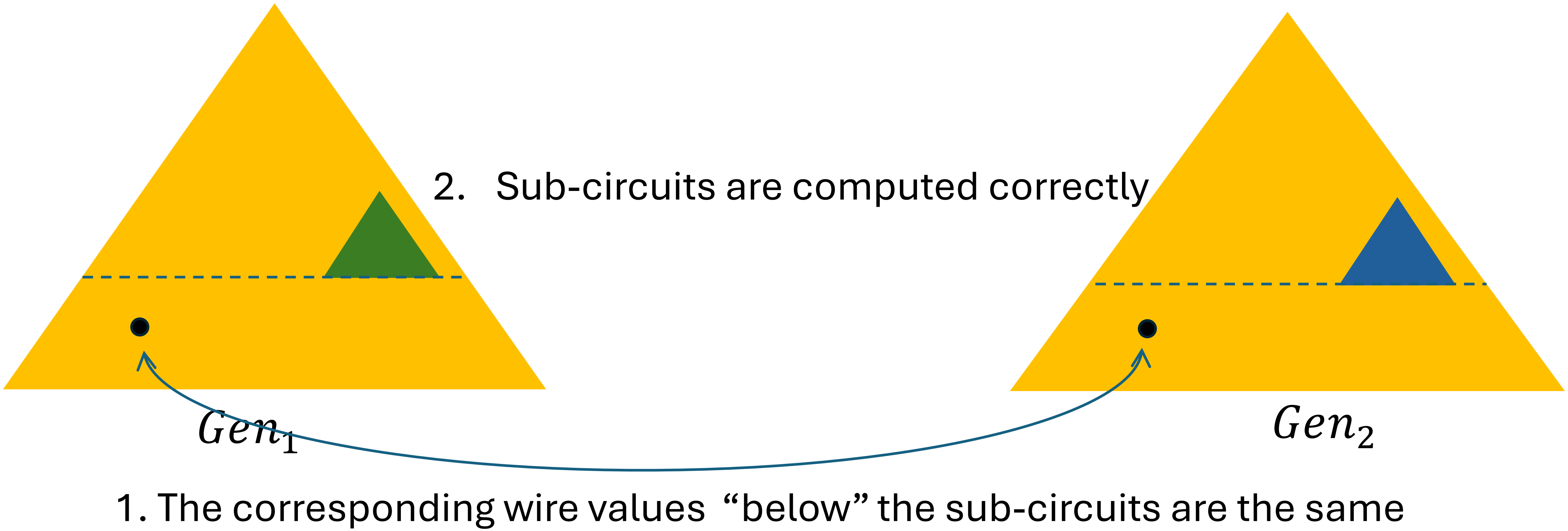
Gen_1 Gen_2

Blueprint of the Proof



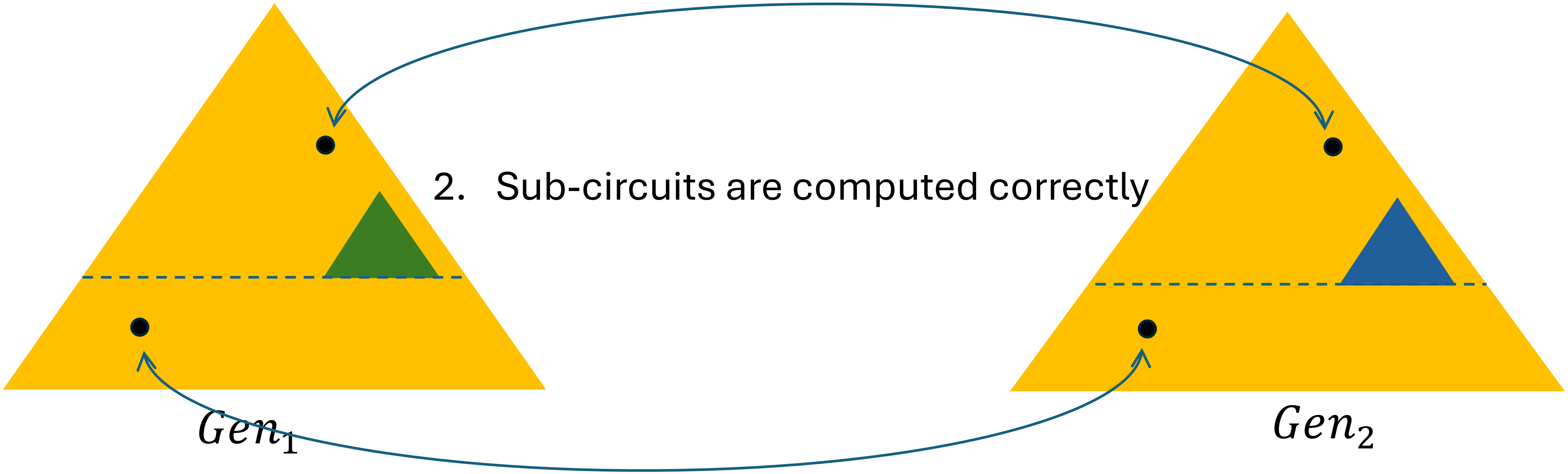
1. The corresponding wire values “below” the sub-circuits are the same

Blueprint of the Proof



Blueprint of the Proof

3. The corresponding wire values “above” the sub-circuits are the same.



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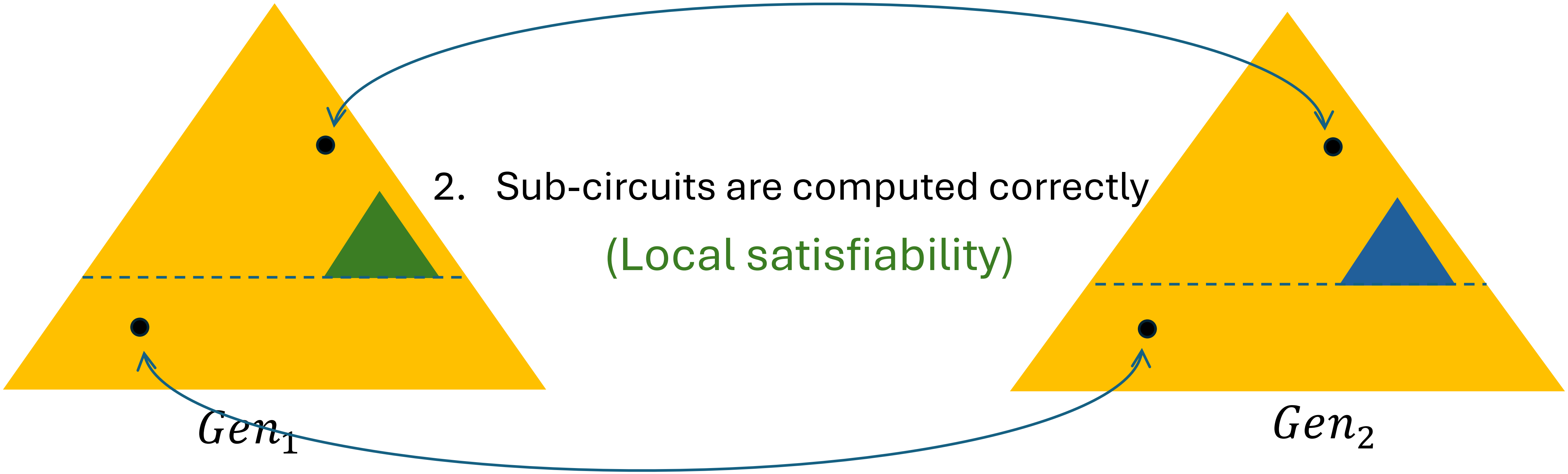

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Proof of Puncturing Argument: Blow the Subckt

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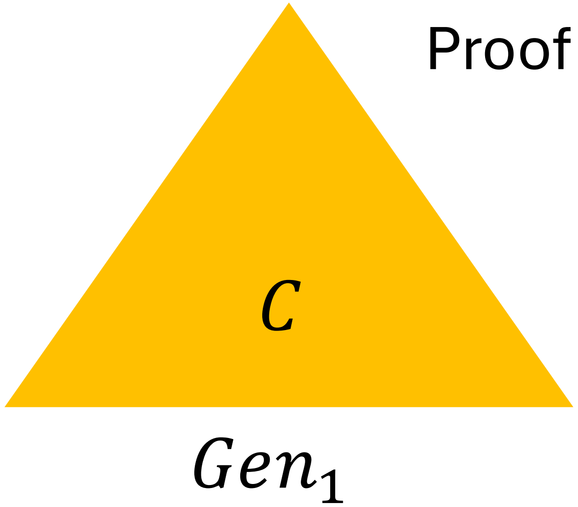
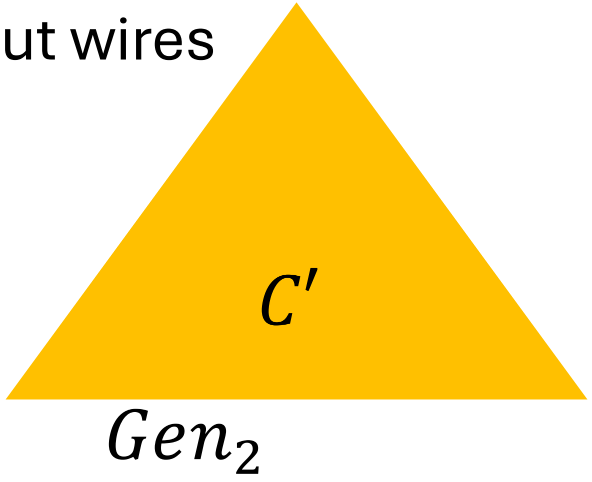
Proof by induction from the bottom, i.e. input wires





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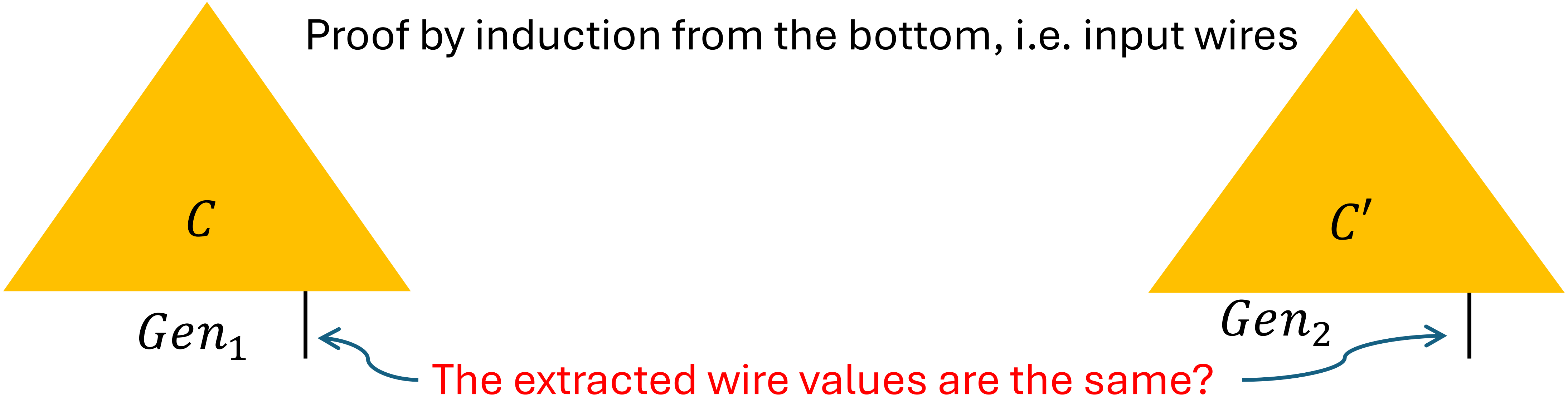

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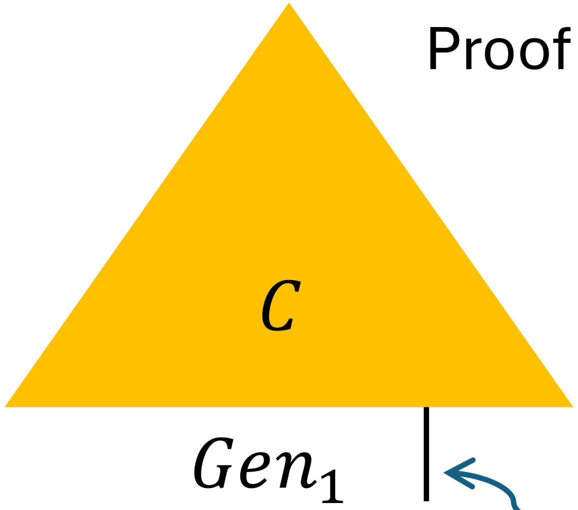

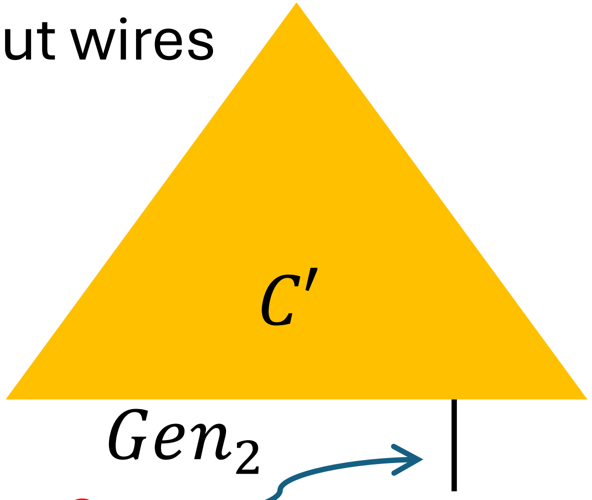

The extracted wire values are the same?

Add Statements that BARG Proves: $\forall i,\ \exists$ local openings of the i-th input wire w.r.t. h_1, h_2, and the wire values $w_i, {w_i}'$ where $w_i = {w_i}'$.

Moving Up in the Layers

: extracted gate outputs in local assignment generators

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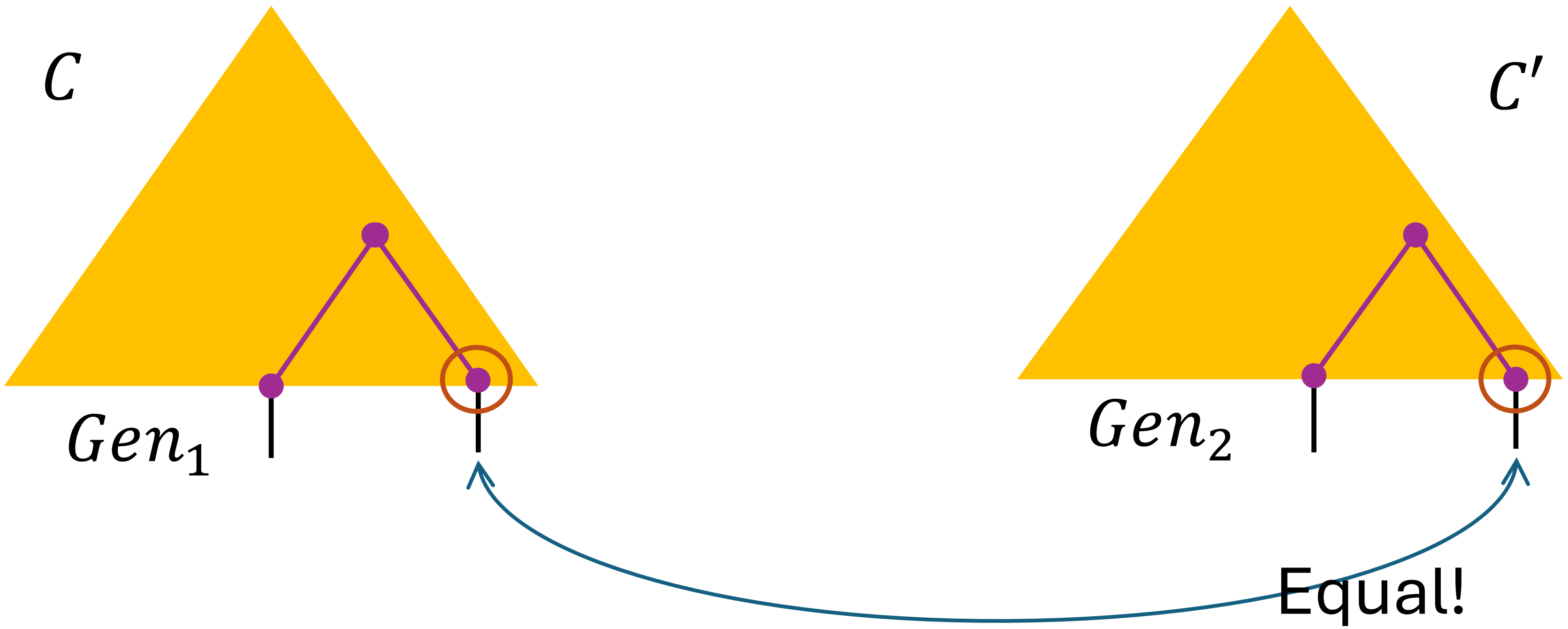

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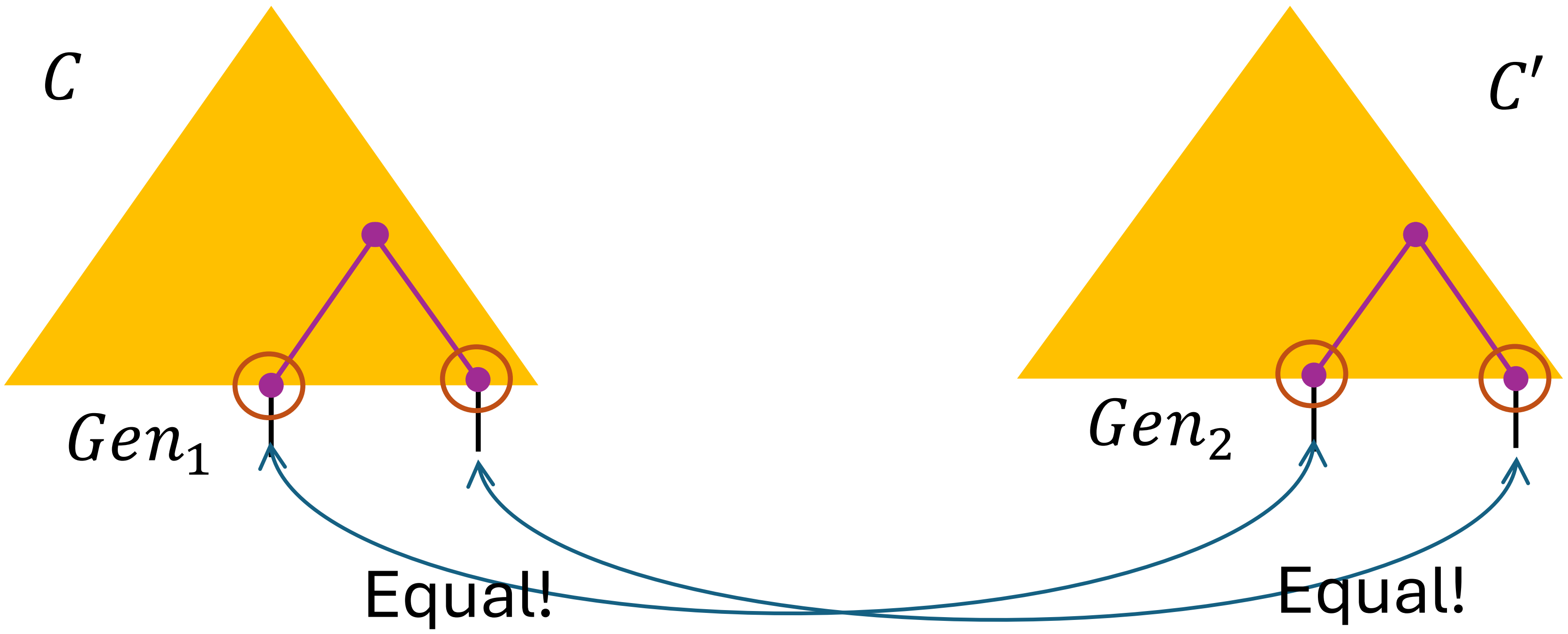

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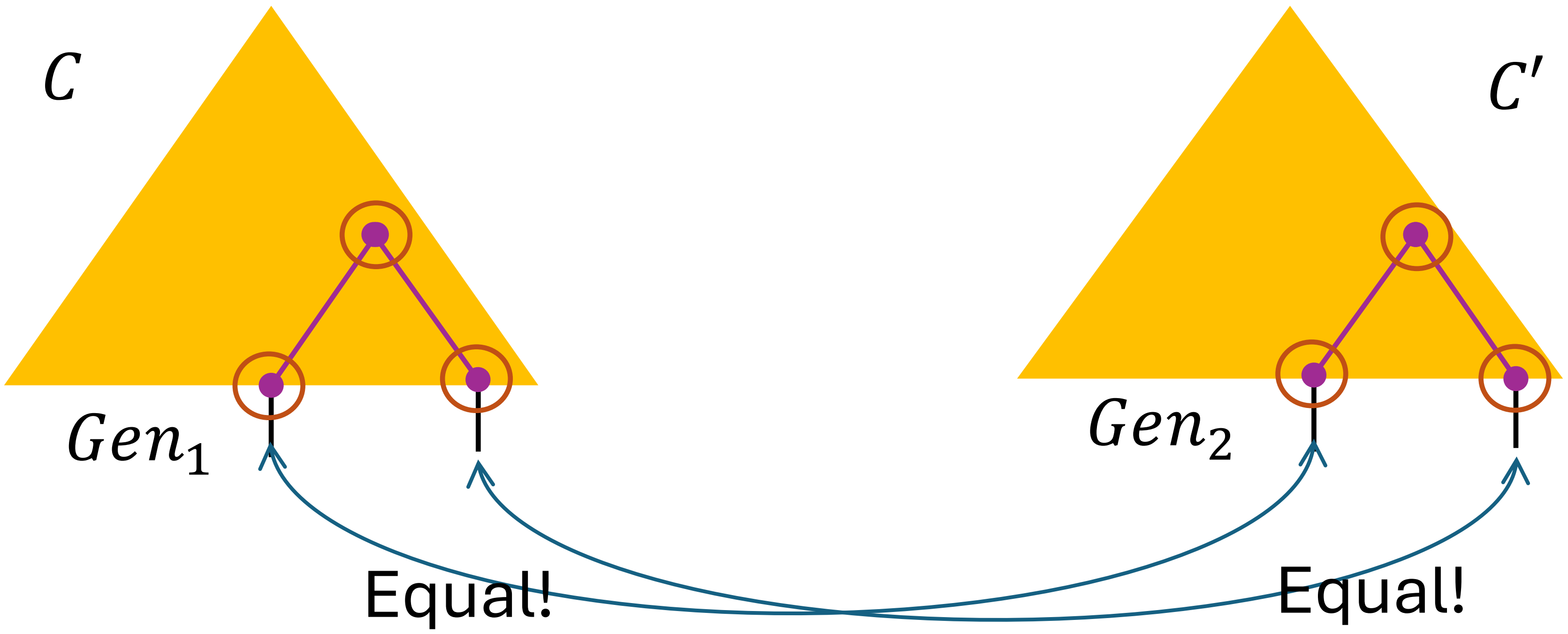

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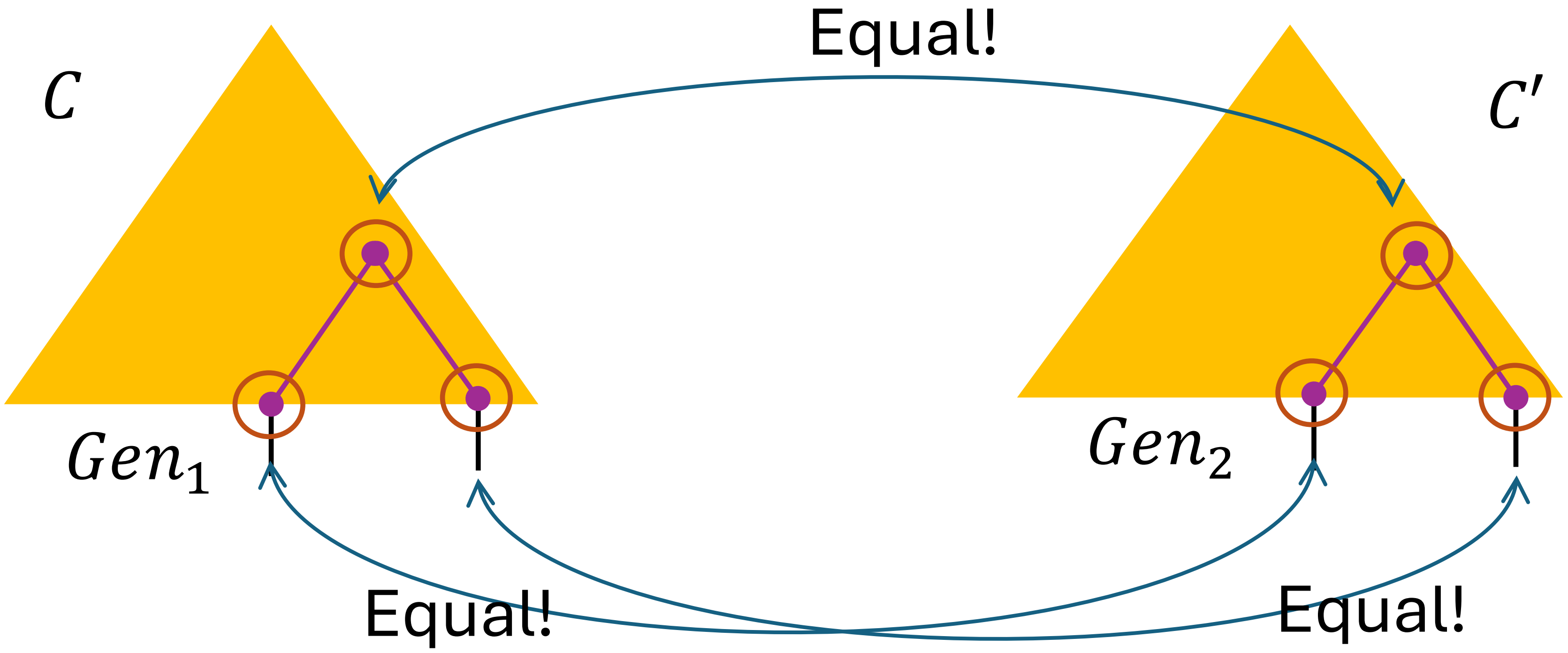

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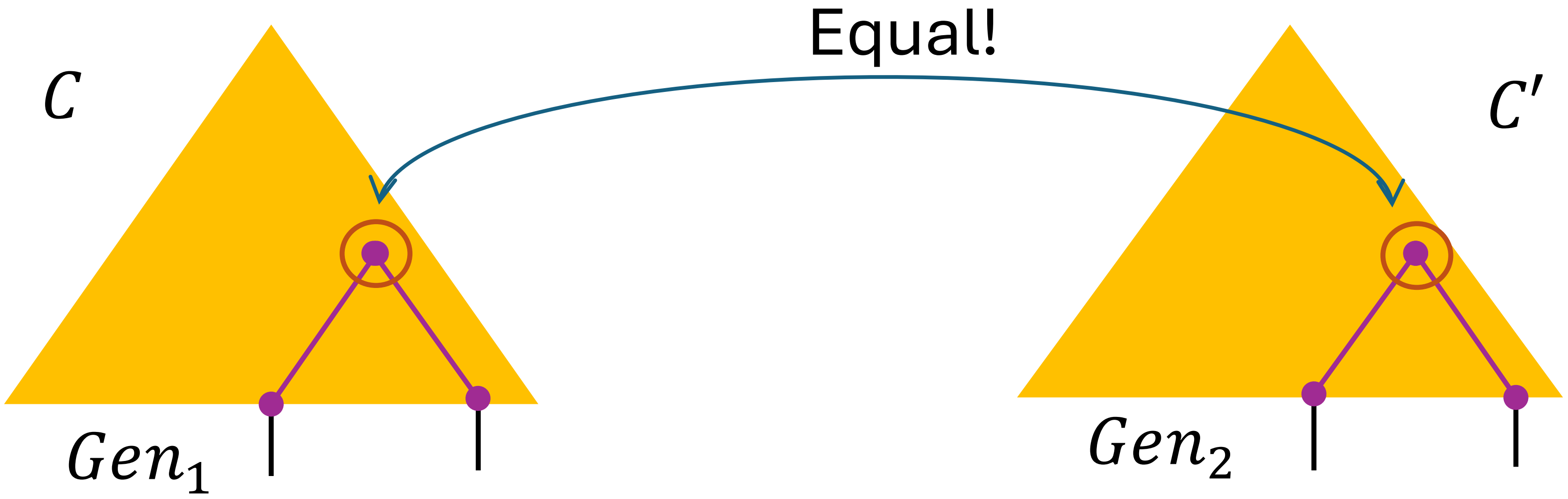

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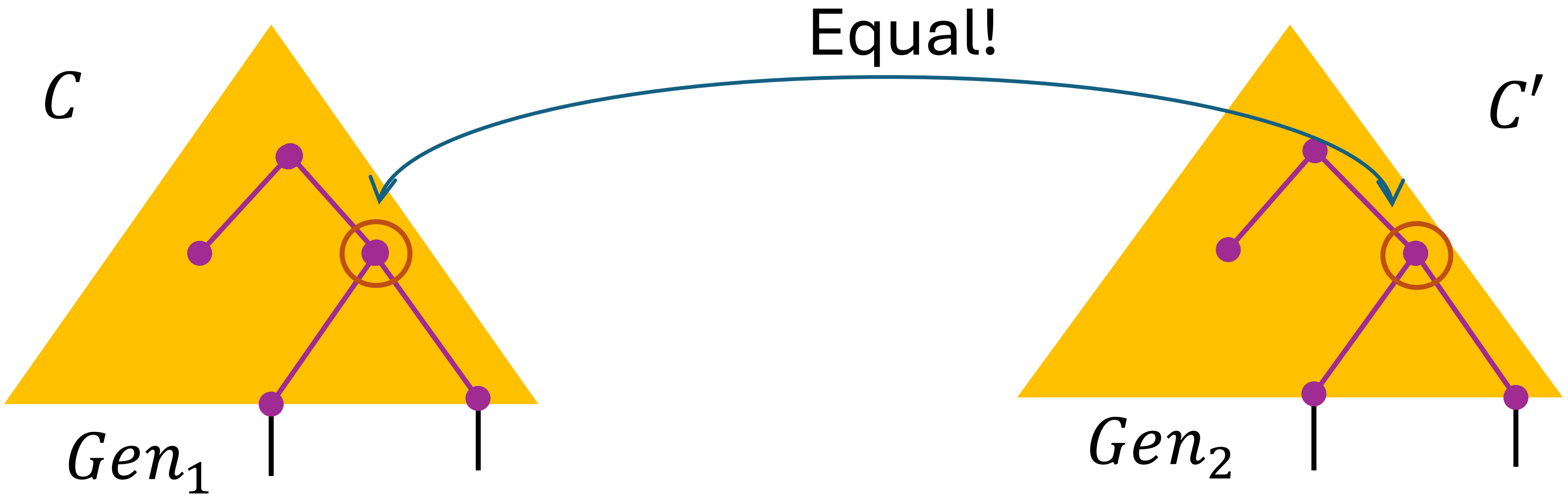

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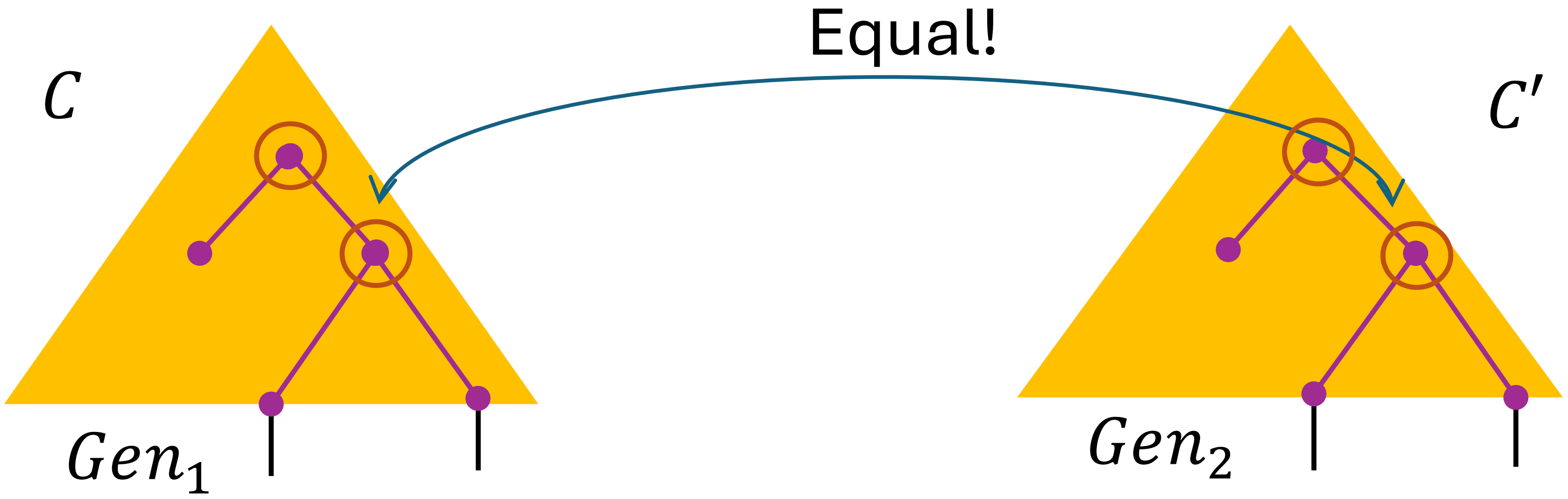

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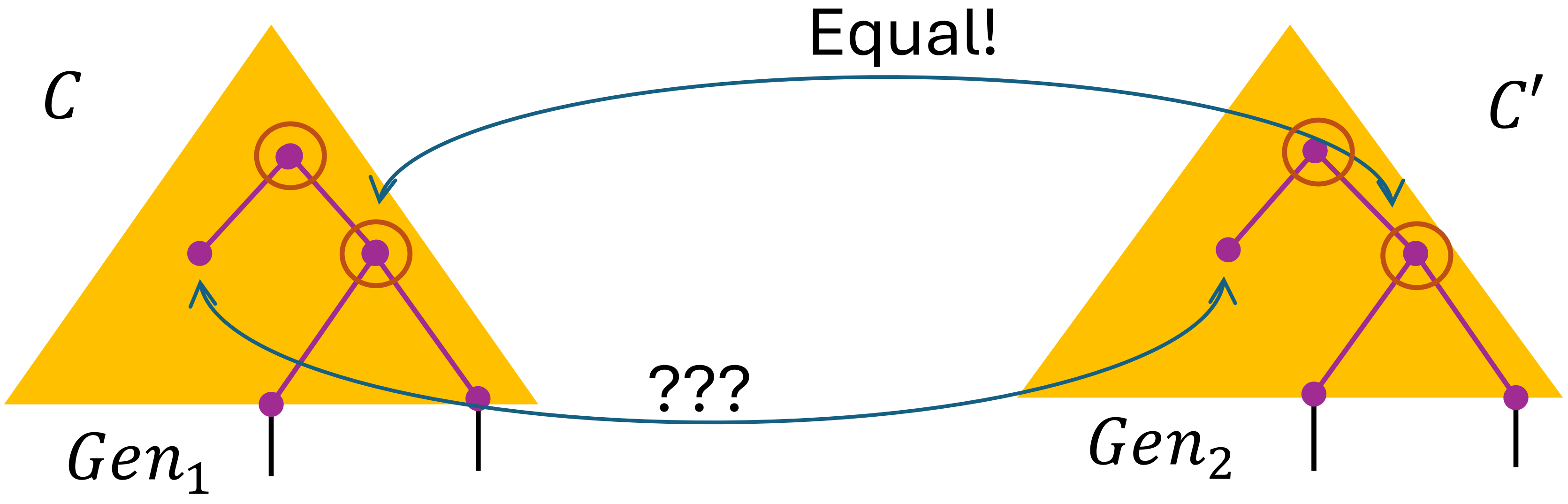
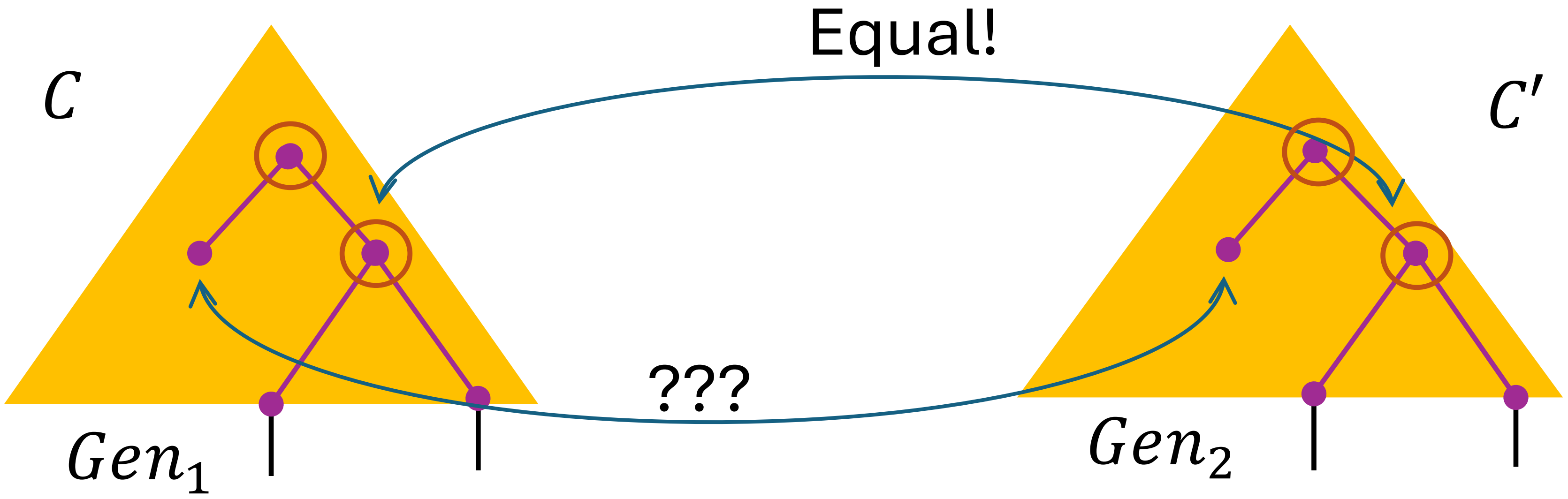


: extracted gate outputs in local assignment generators

Moving Up in the Layers

We need to extract their children first…

and the children of their children first…



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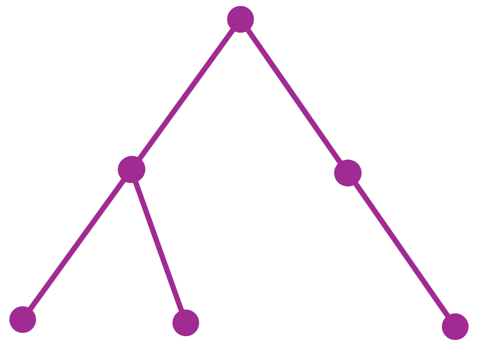
Abstracting as a Pebbling Game



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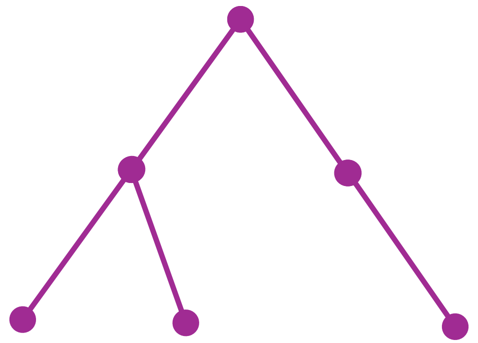

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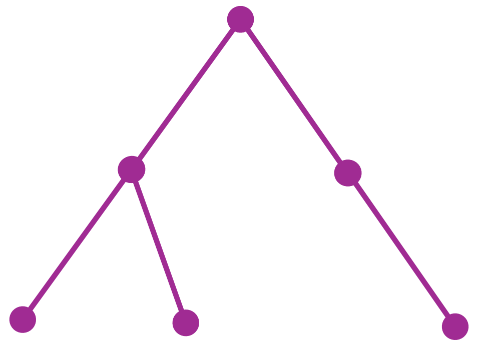

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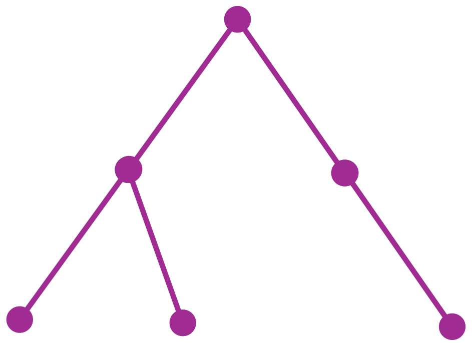

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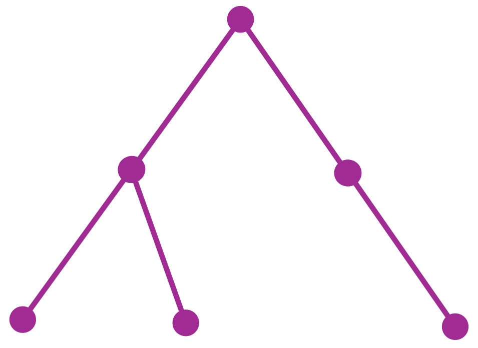

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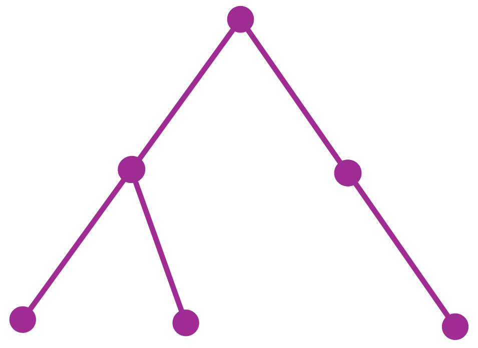

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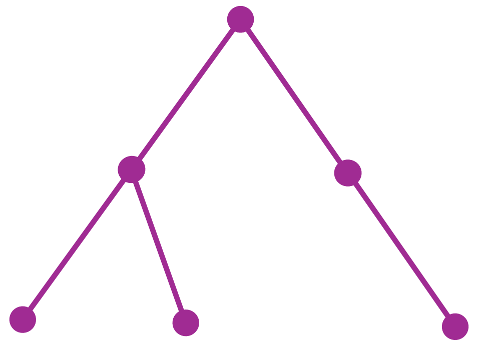

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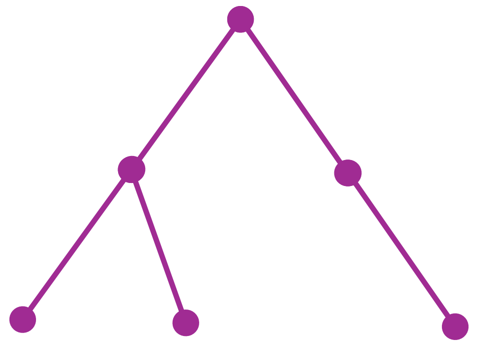

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Reduction time: 2^{depth}! More efficient reduction?

Augmenting Ckts with Collision-Resistance Hashes

: extracted gate values in local assignment generators

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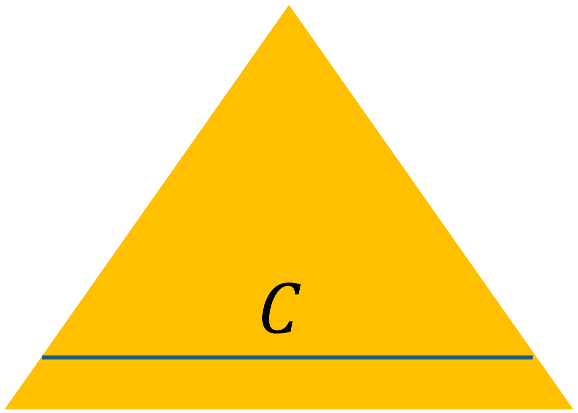
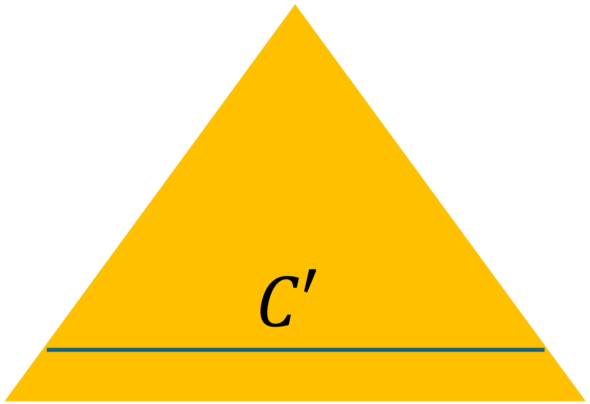
Merkle hash the gate values at each layer via Collision-Resistance Hash (CRHF)





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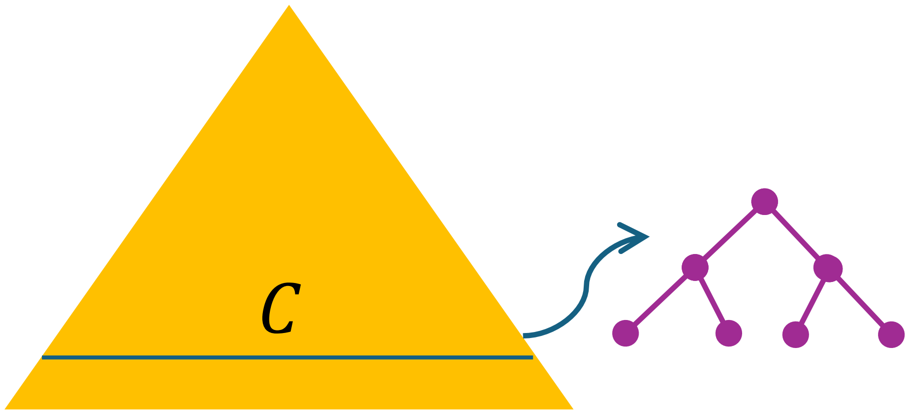

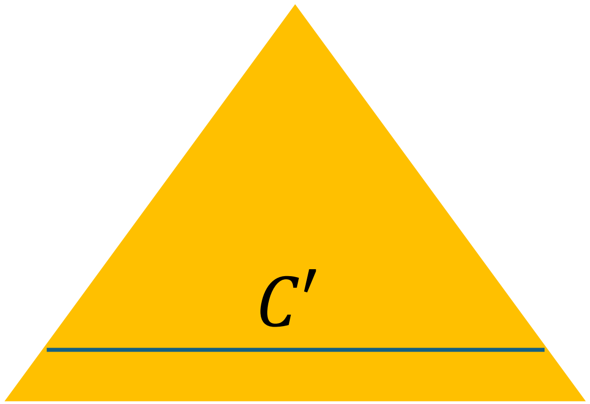

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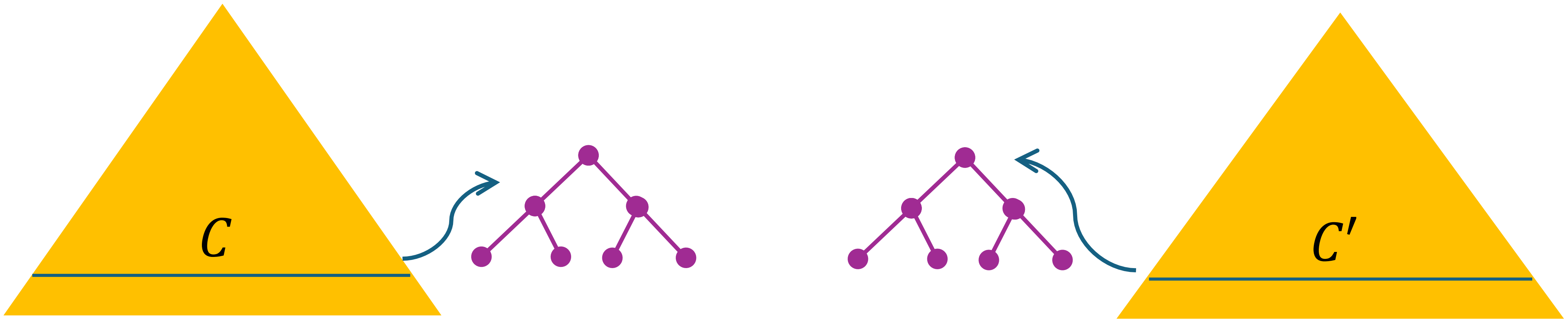

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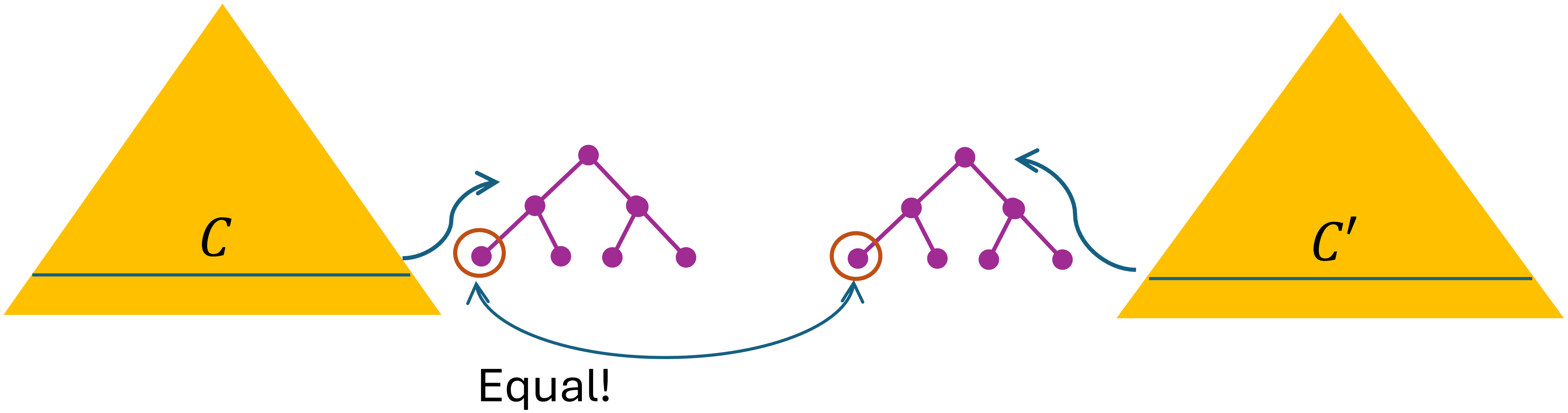

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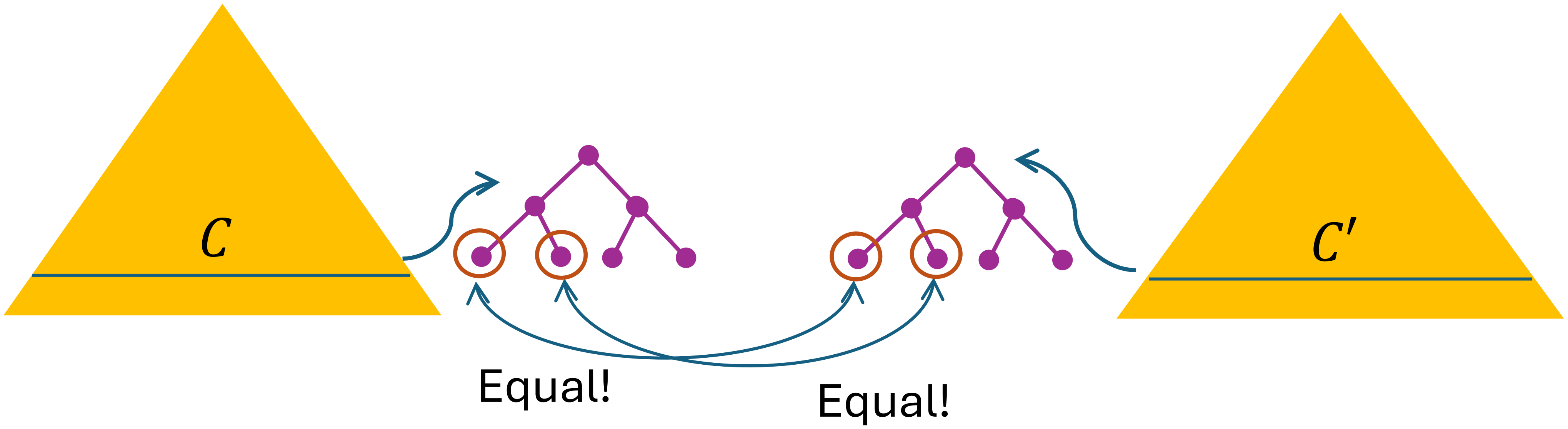

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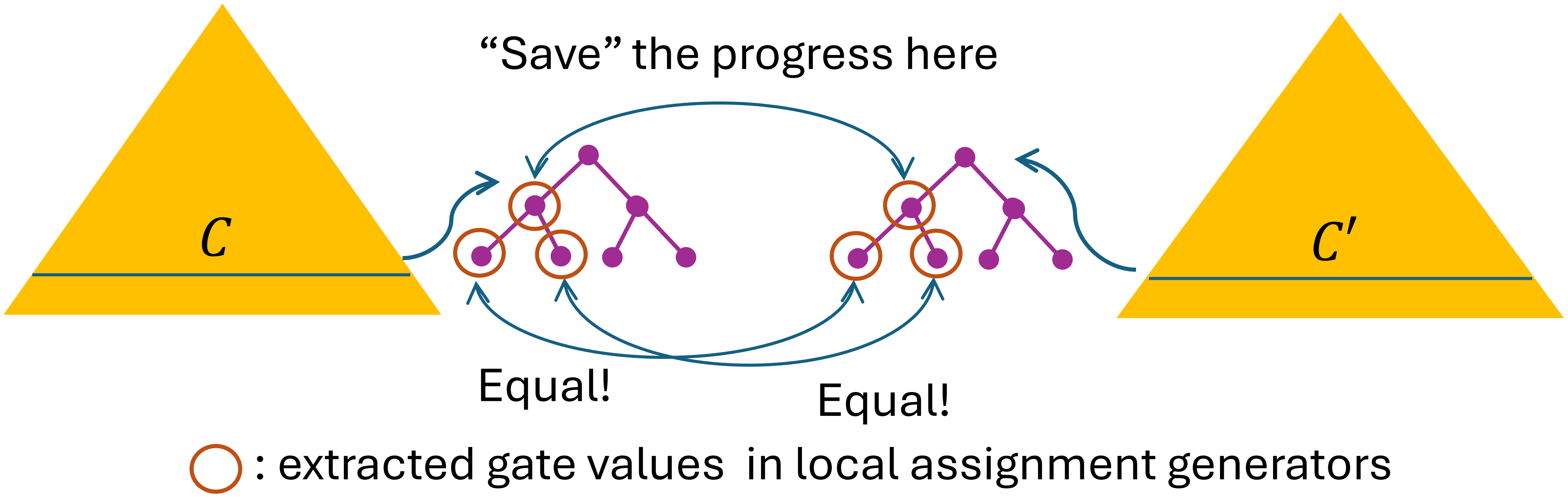

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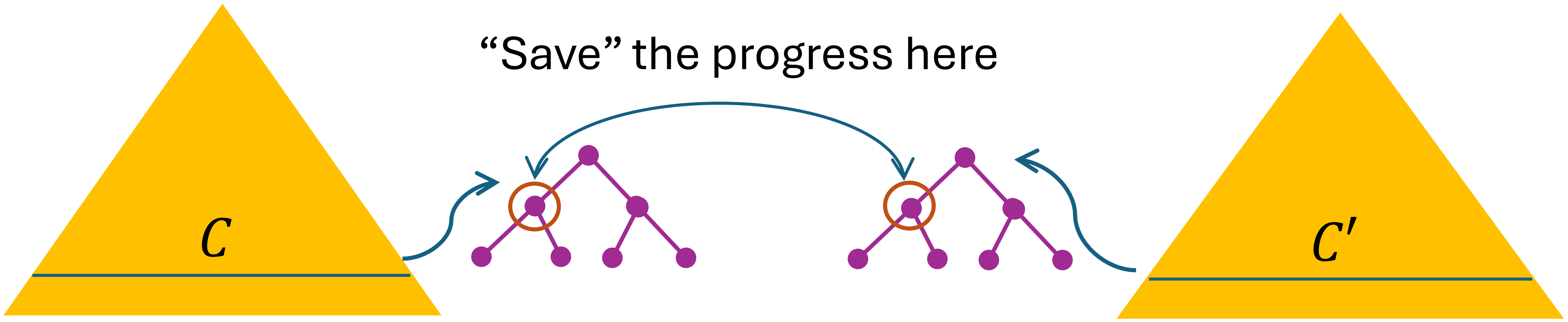

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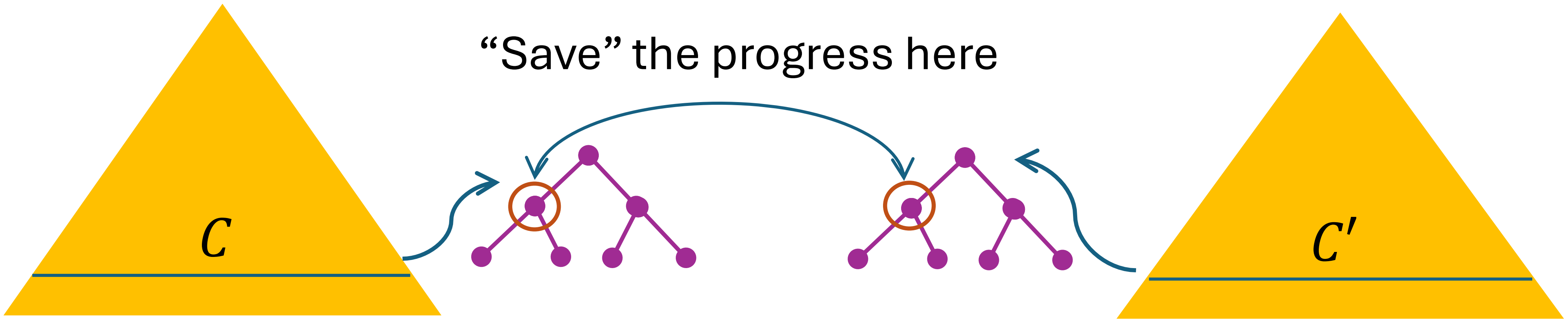

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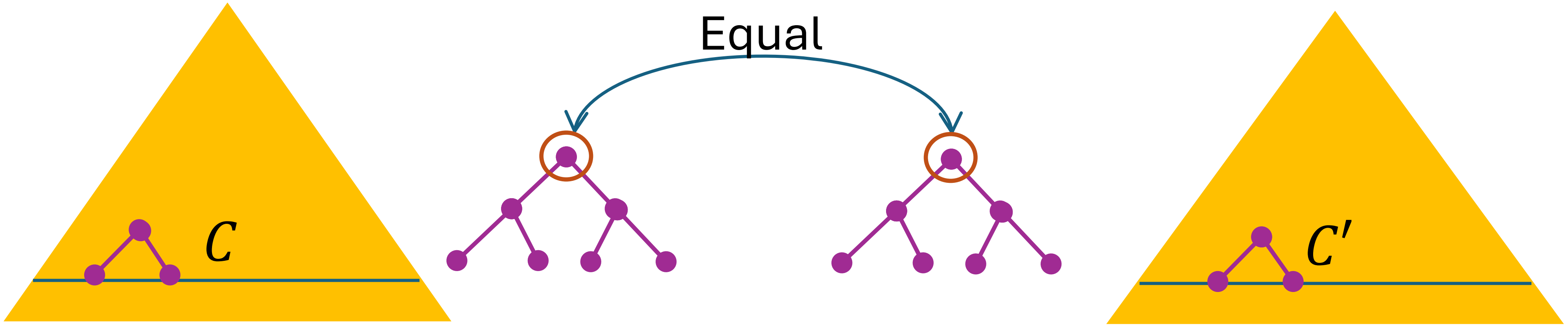
The entire layer is “equal” $\Longleftrightarrow$ the roots of CRHF trees are “equal”.

Moving Up in the Layers



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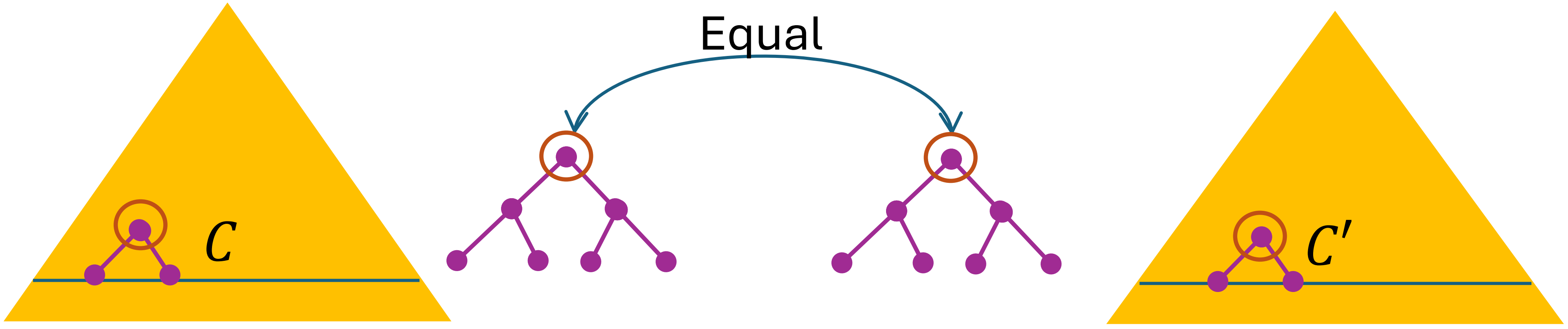

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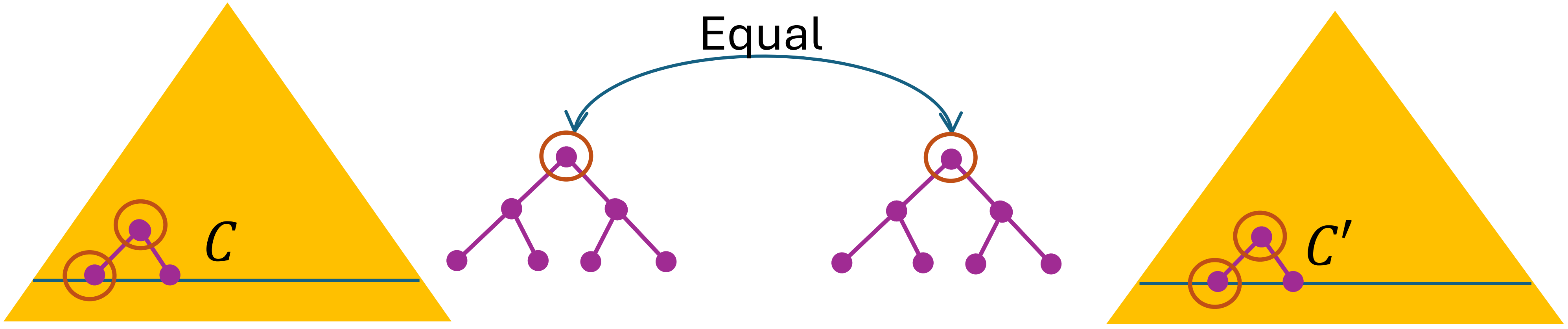

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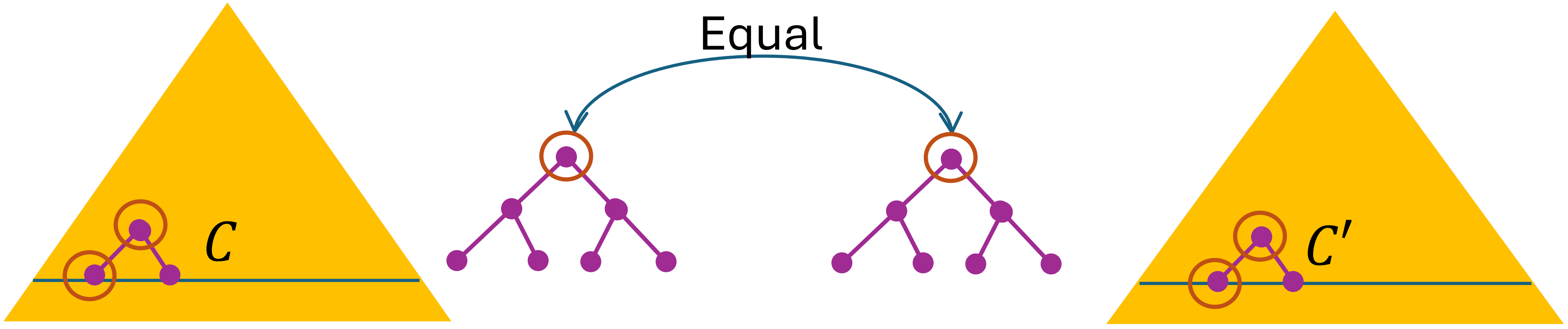
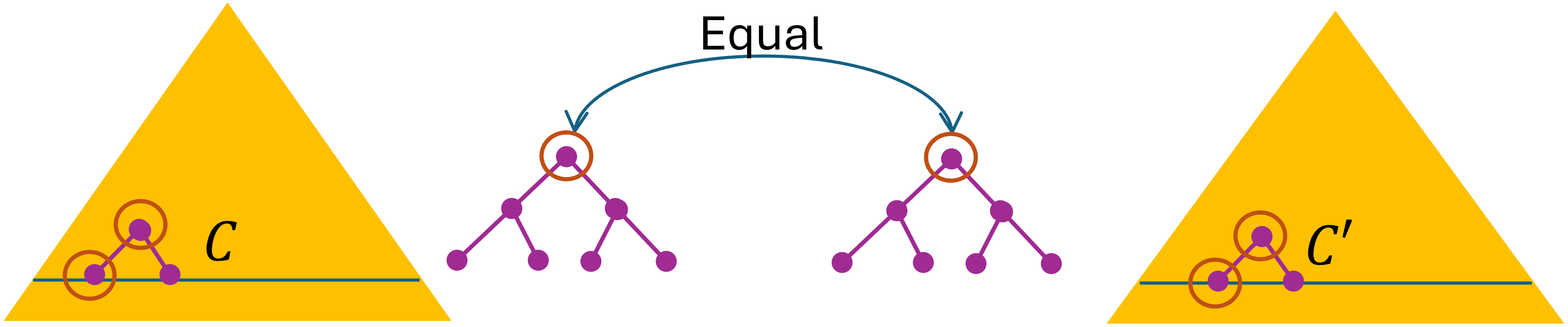


Suppose the left child is the 2nd node in the layer

: extracted gate values in local assignment generators

Moving Up in the Layers

"Load" the saved progress from the roots



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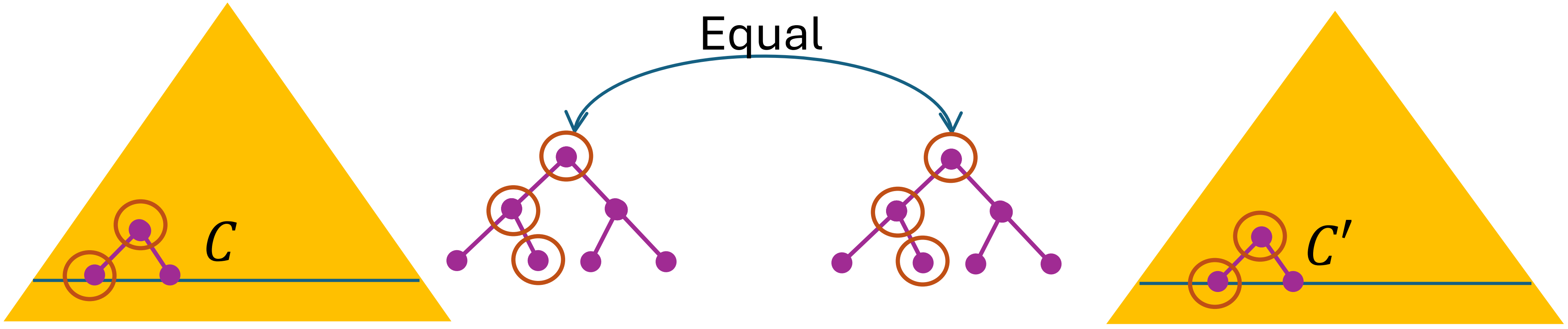

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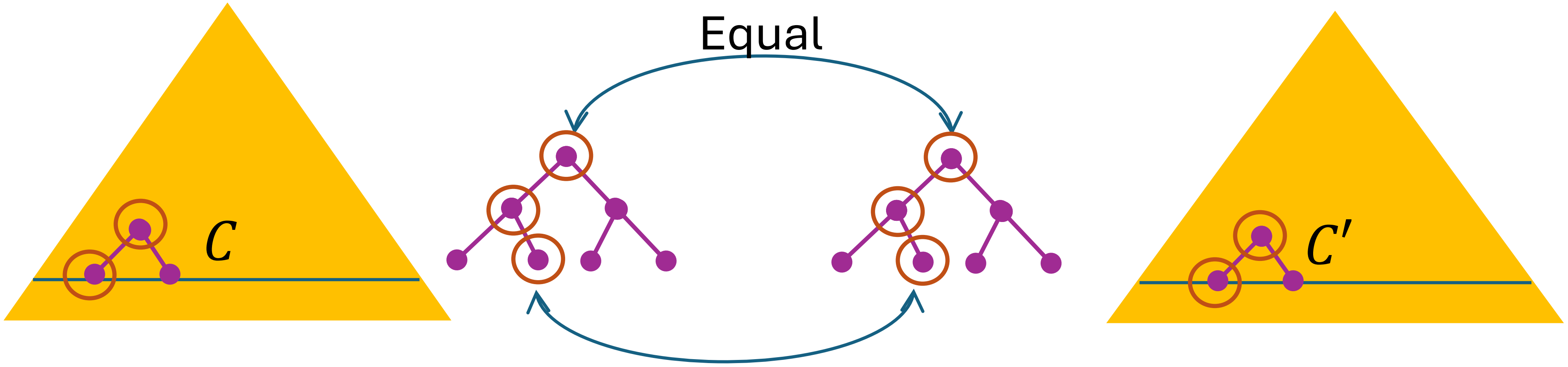

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◯ : extracted gate values in local assignment generators

Save the Progress in the Middle of the Subckt

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Merkle-Hash the layer, but “puncture” the-root-to-leaf path for leaves in subcircuit, and use the roots of remaining subtrees to save the “equality” progress so far.

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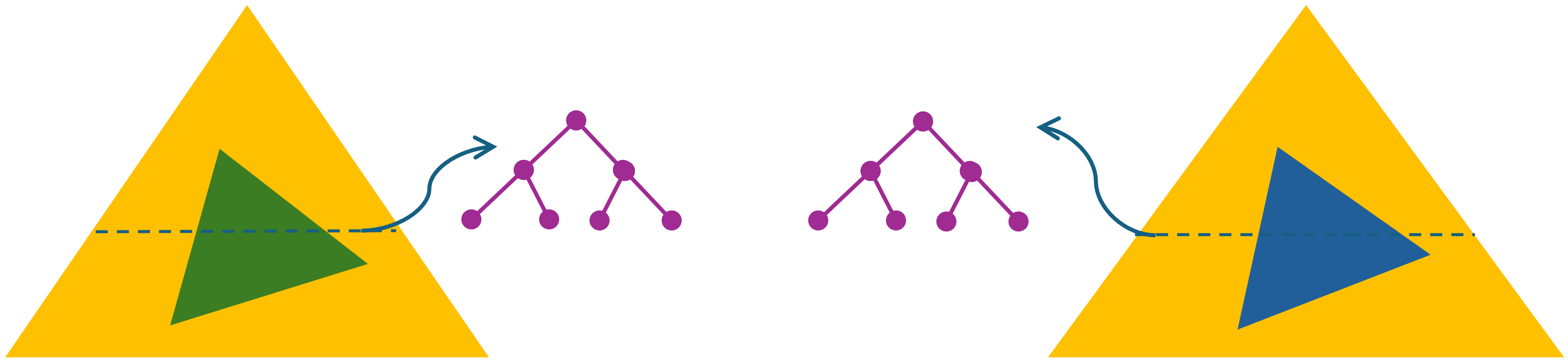

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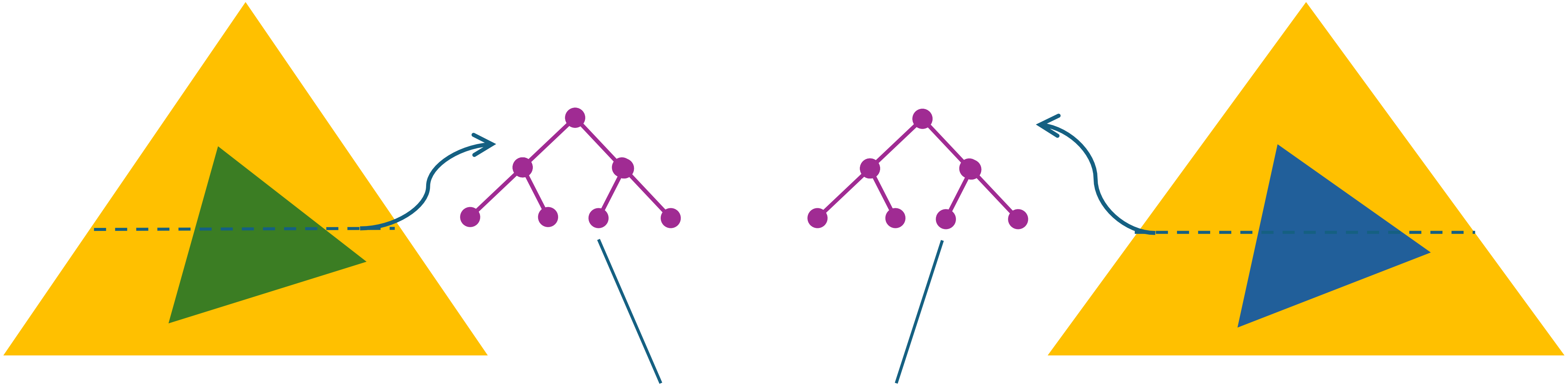

e.g. 3rd leaf is in the subcircuit

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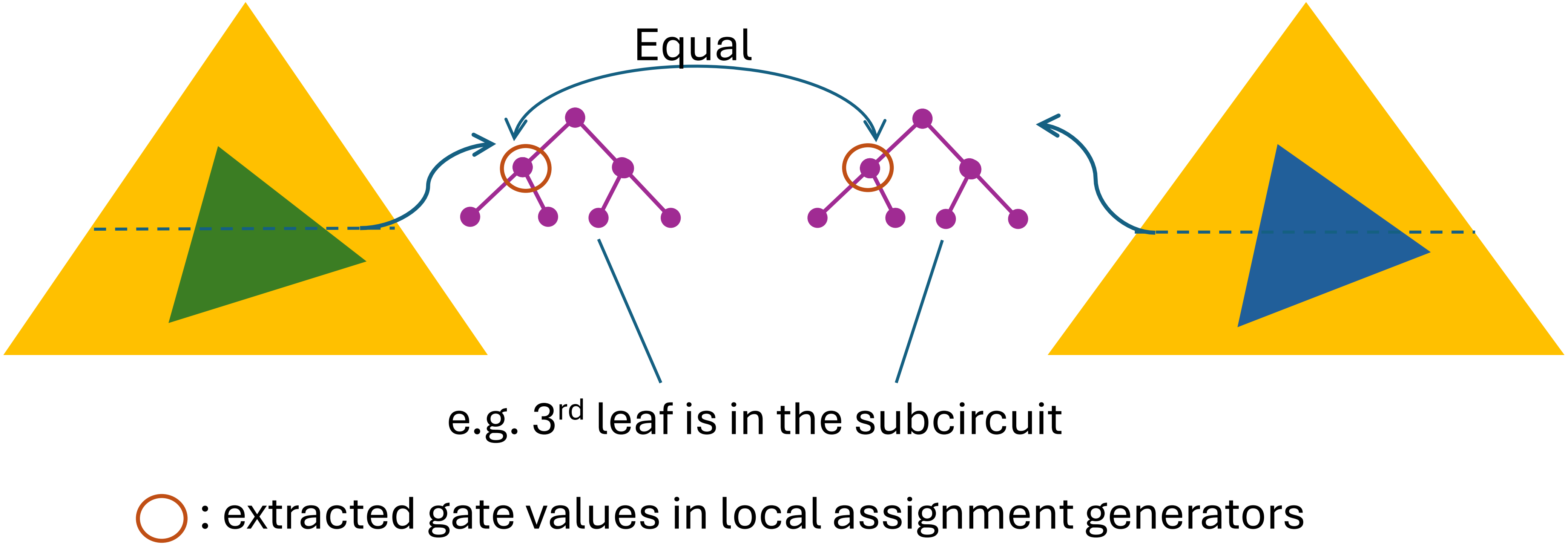

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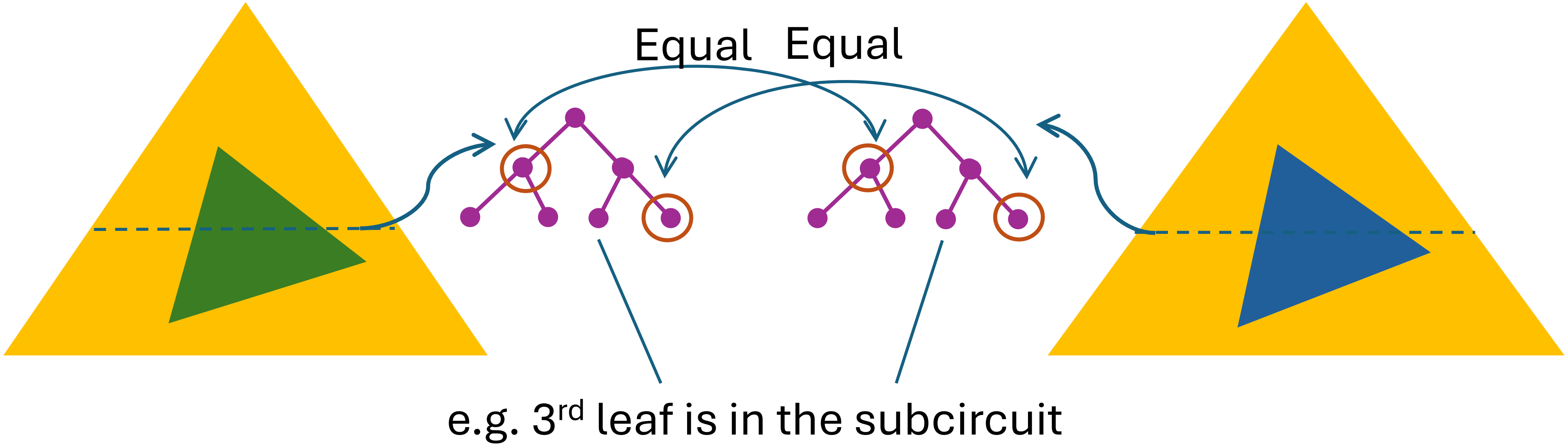

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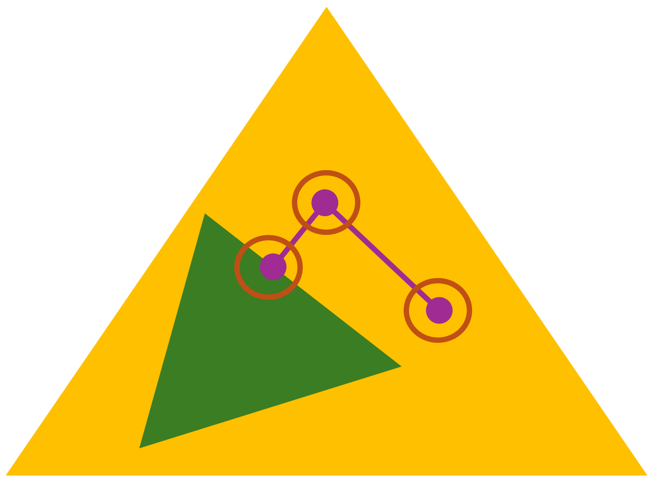
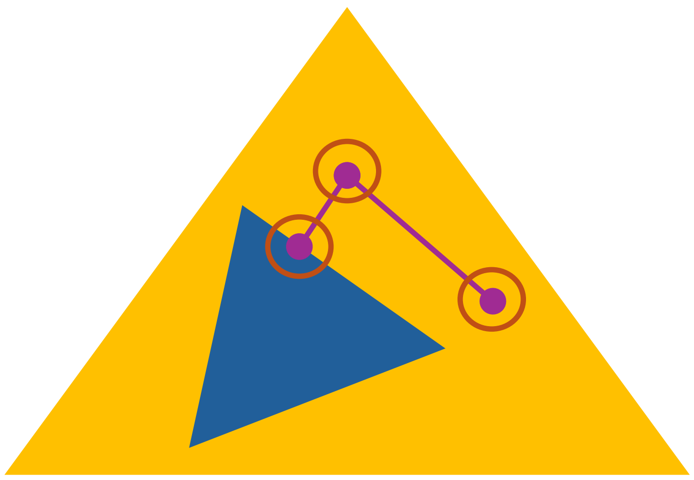
Finishing the Proof: “Above” the Subckts





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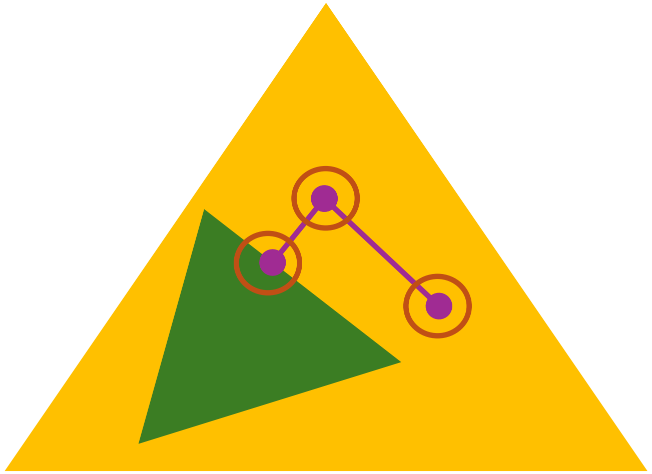
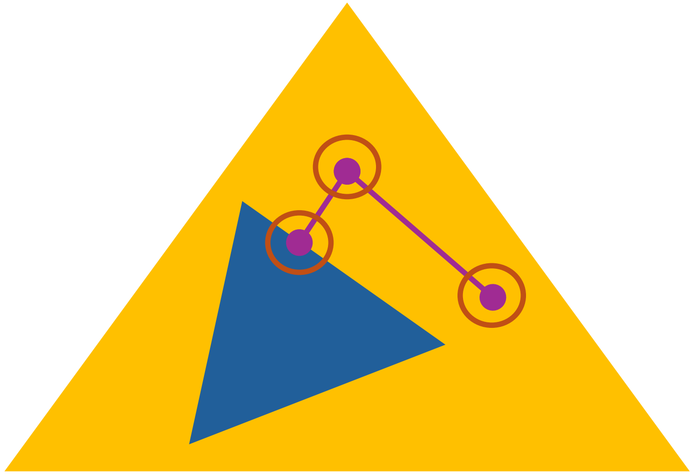
Finishing the Proof: “Above” the Subckts

A general gate: Left Child: an output wire of the subcircuit
Right Child: a wire outside of the subcircuit





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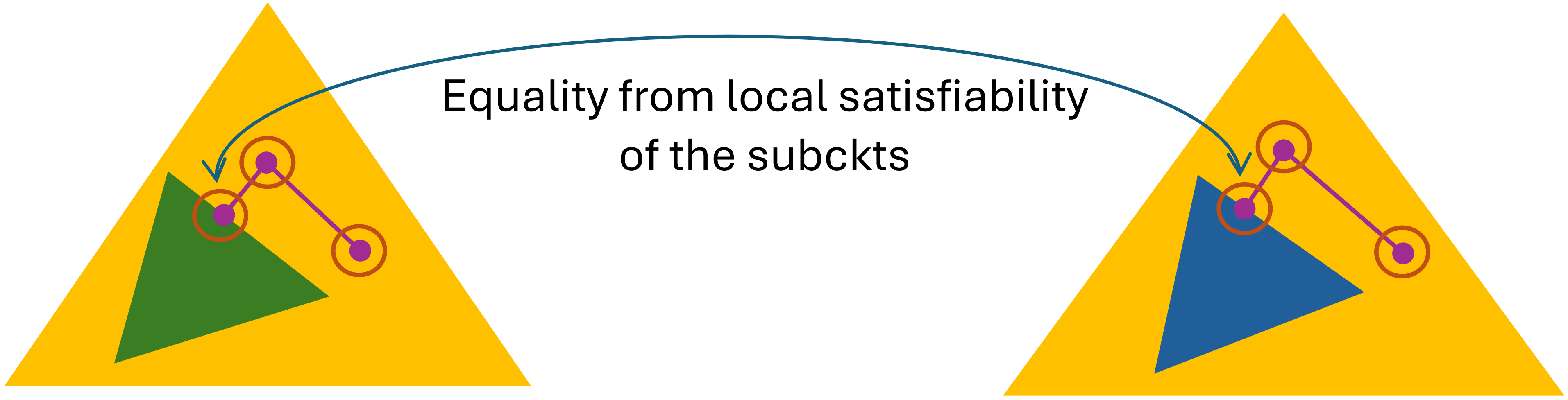

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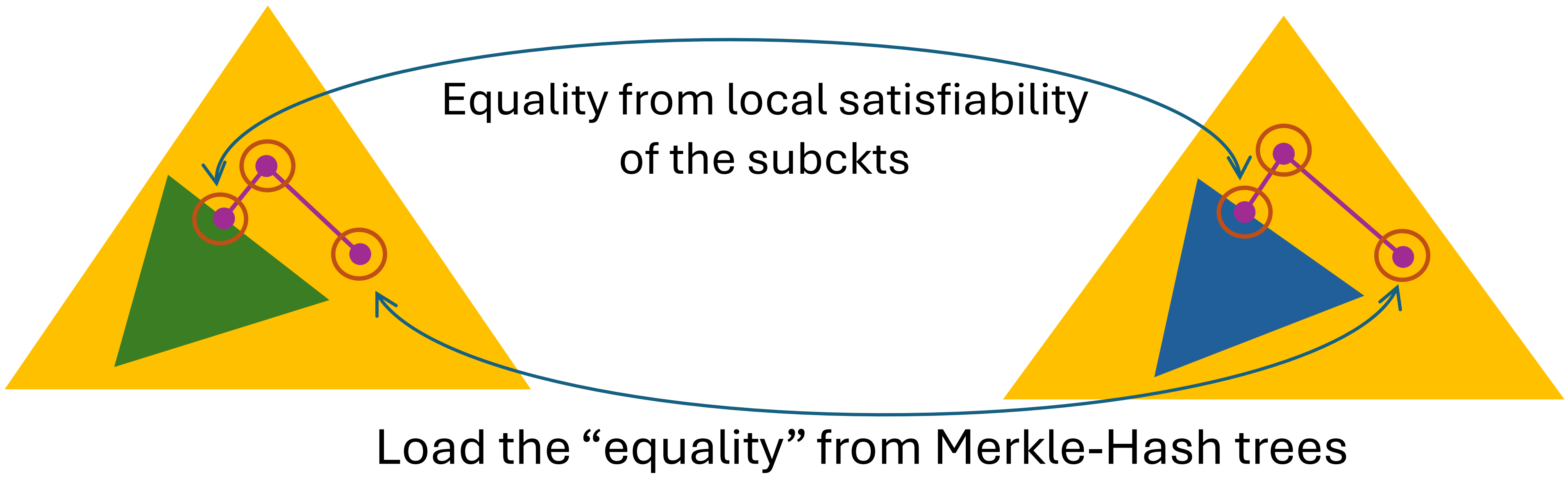

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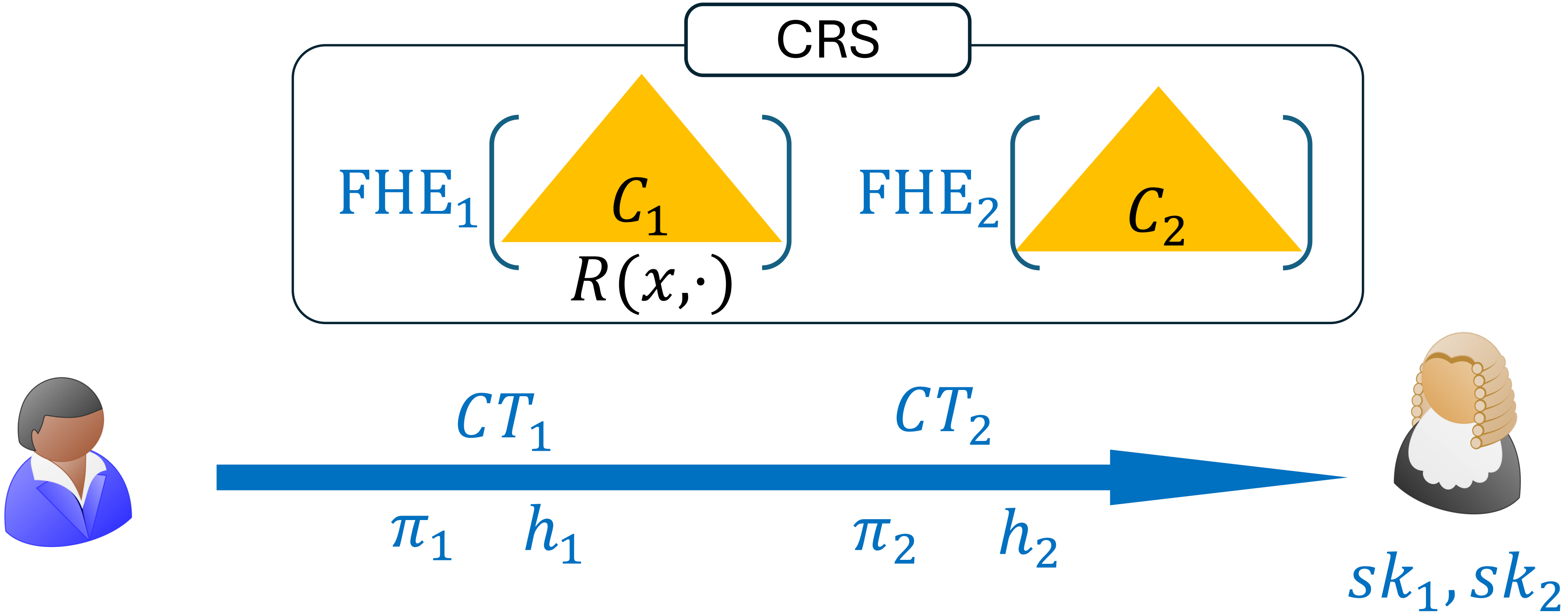
Rest of the Talk

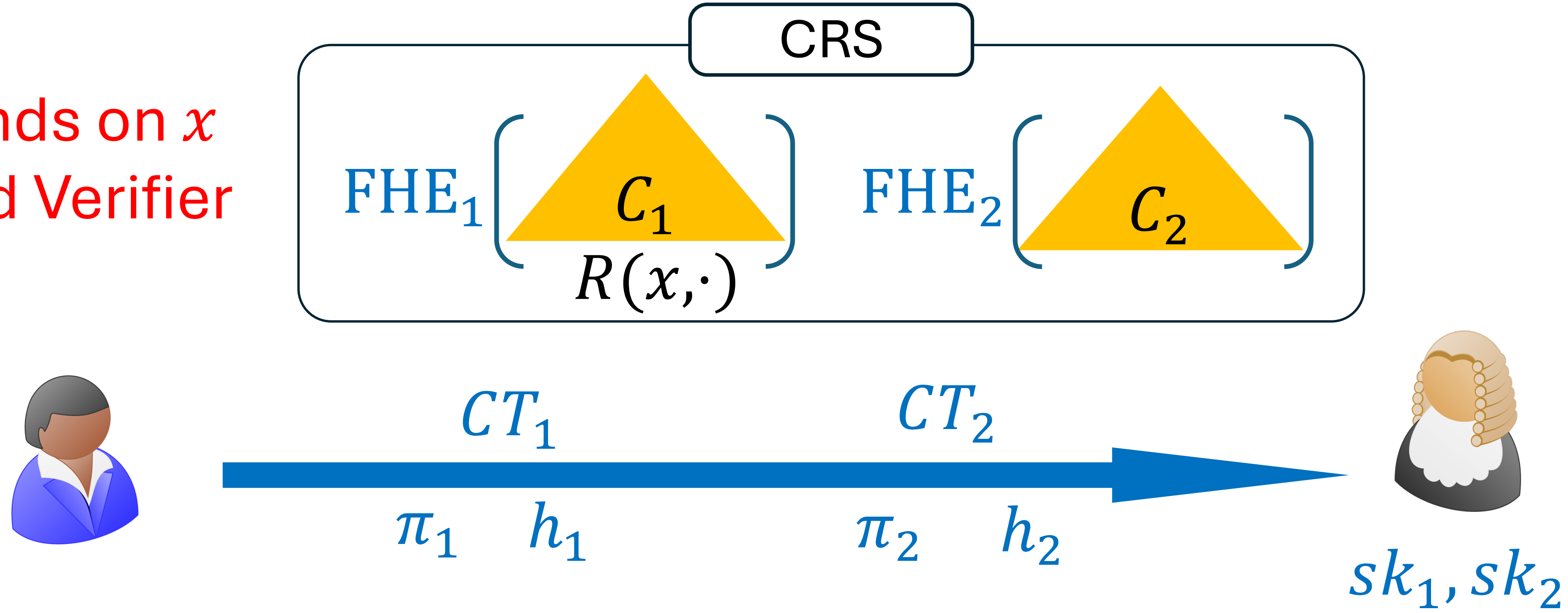
- **Achieving Public Verification & Random CRS**
- Discussion

Construction So Far



Construction So Far

- CRS depends on x
- Designated Verifier



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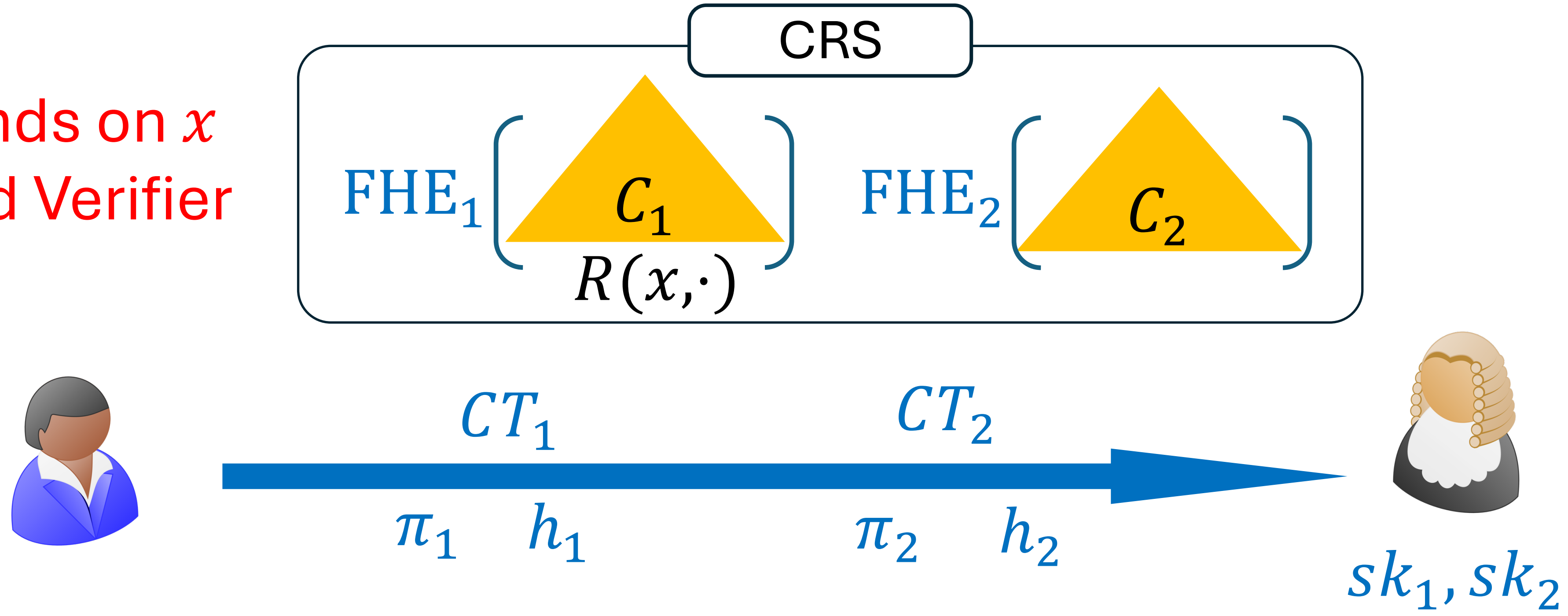

Public Verification: give out sk_1?

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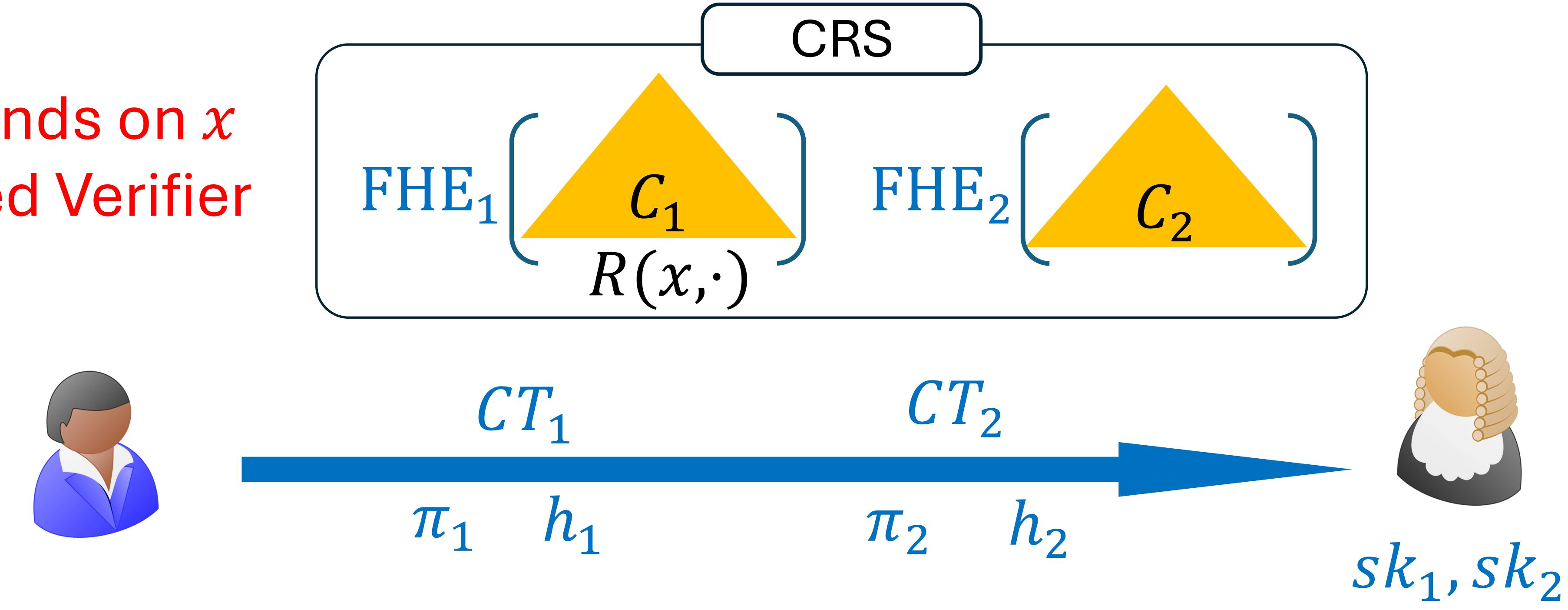
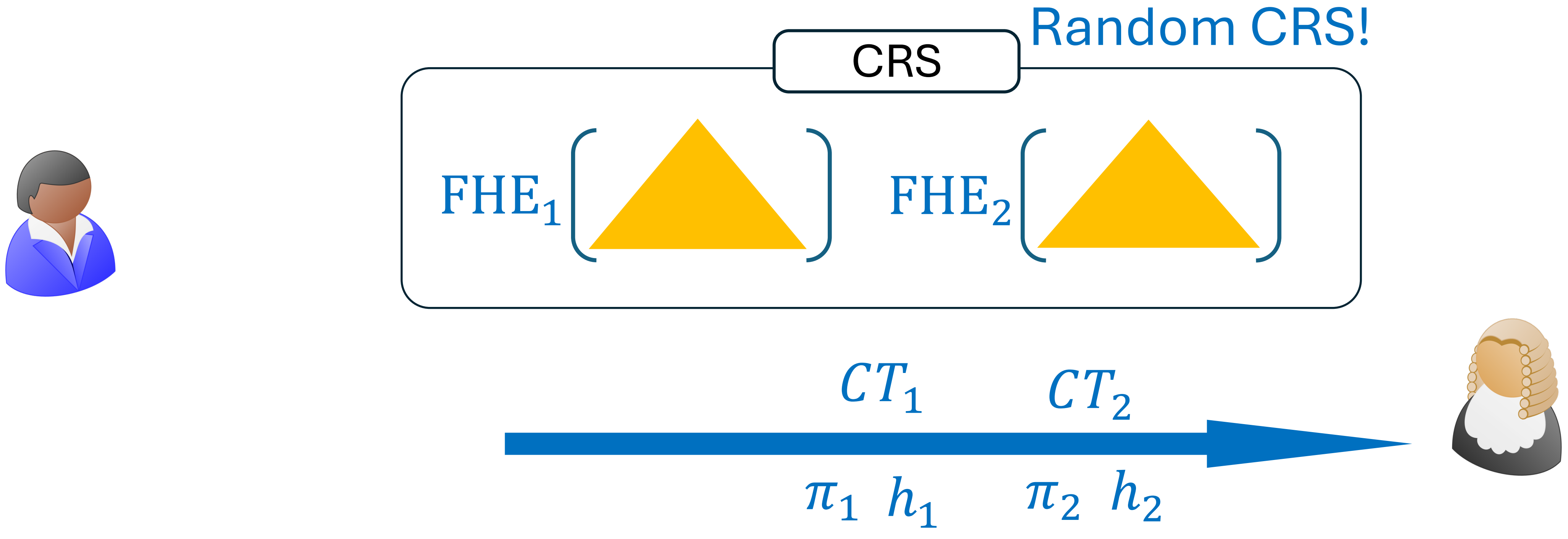


Public Verification: give out sk_1?

We don't need to encrypt $R(x,\cdot)$!

Achieving Public Verification



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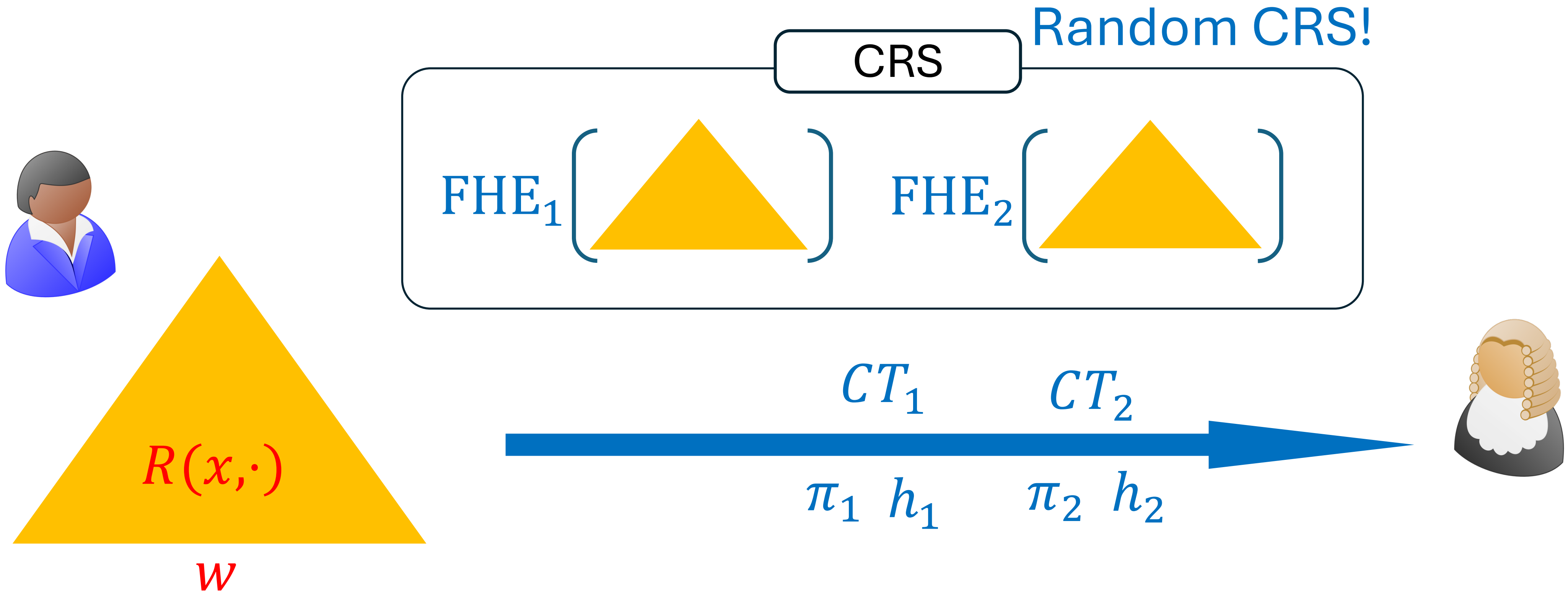

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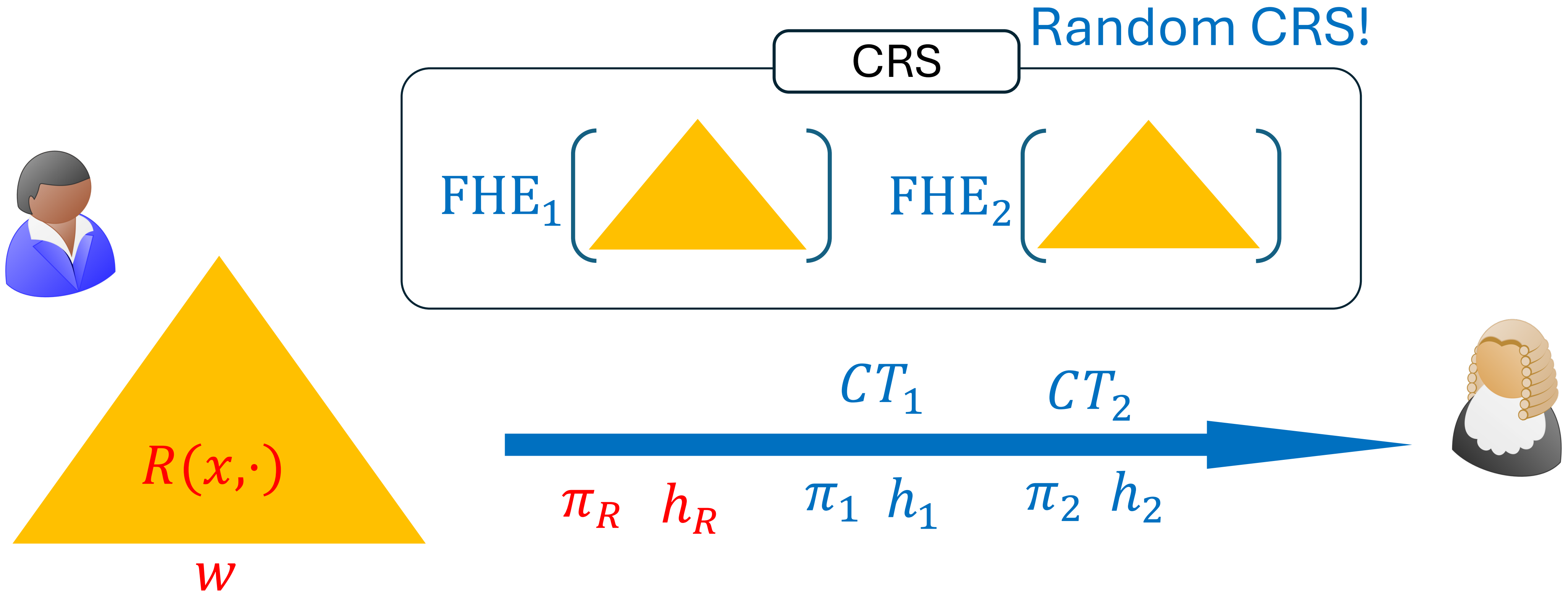

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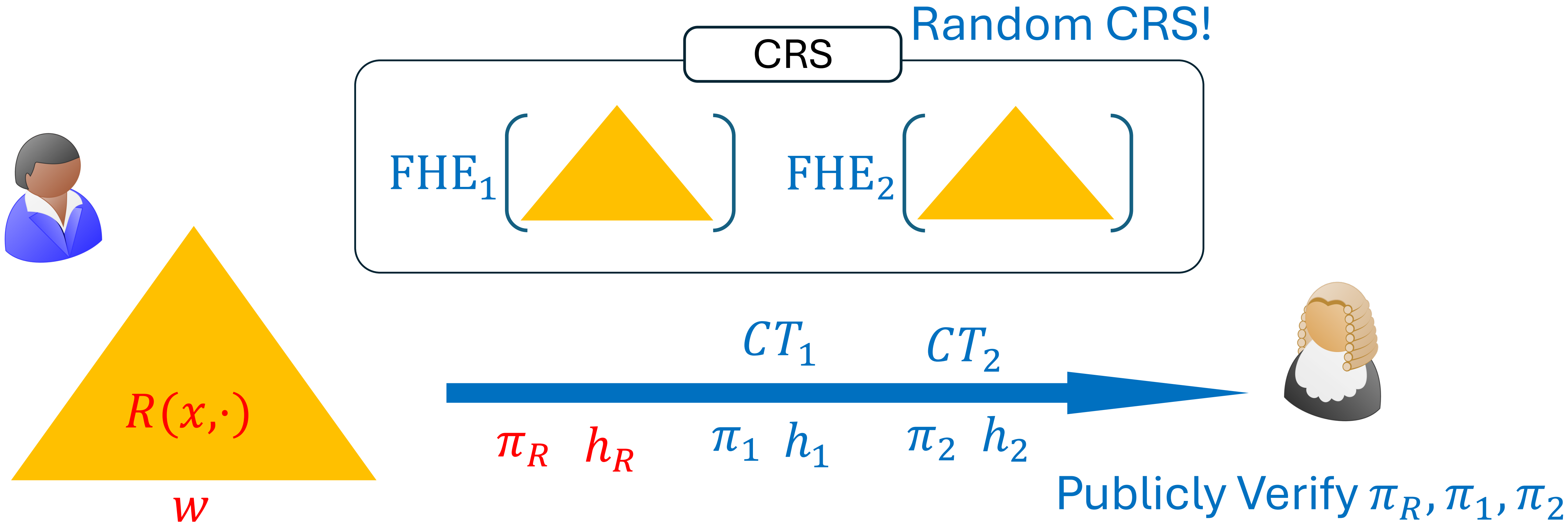

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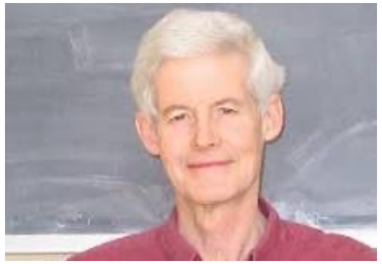
Discussion

(Perspective on this line of ‘logic-based’ approach)

Recall: **Duality** between Logic and Computation

Proof Complexity	Computational Complexity
Frege	Formulas
Extended Frege	Circuits
Poly-size Extended Frege	Poly-size Circuits
Cook’s Theory PV	Poly-time Turing Machines
…	…



Logic: A Forgotten Structure in Cryptography?

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Non-black-box use of functions, i.e. use their circuits

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many examples:
FHE, garbled circuits, arithmetization,
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Impossibility?

Black-box separations

Thank you!

Q & A