Multi-key Fully-Homomorphic Encryption in the Plain Model

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University of California, Berkeley

Multi-key Fully-Homomorphic Encryption [LTV12]

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Public Keys: $\mathbf{pk}_1$ $\mathbf{pk}_2$ $\cdots$ $\mathbf{pk}_N$

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Ciphertexts:	m_1	m_2		m_N

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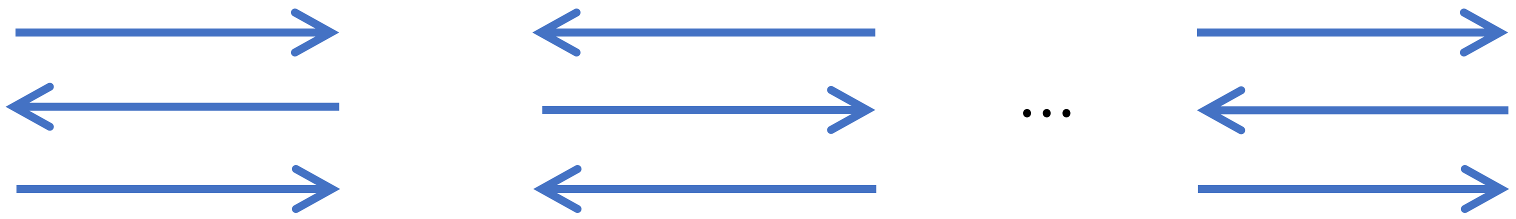
Decryption Protocol:	
Recovery:	$C(m_1, m_2, \ldots, m_N)$

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Decryption Protocol: …

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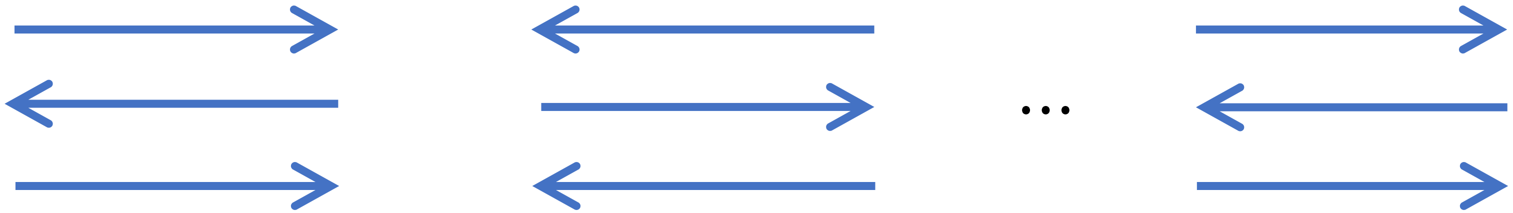

Recovery: $C(m_1, m_2, \dots, m_N)$

- **Security:** adversary can learn nothing beyond $C(m_1, m_2, \dots, m_N)$.

Multi-key Fully-Homomorphic Encryption [LTV12]

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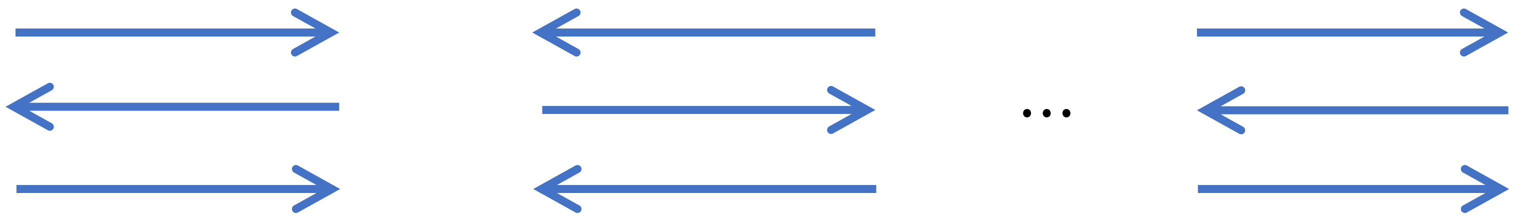

$\cdots$

Recovery: $C(m_1, m_2, \dots, m_N)$

- (Implicit) **Reusability**: decryption can run for different $C(m_1, m_2, \dots, m_N)$ without re-generating the public keys/ciphertexts.

Multi-key Fully-Homomorphic Encryption [LTV12]

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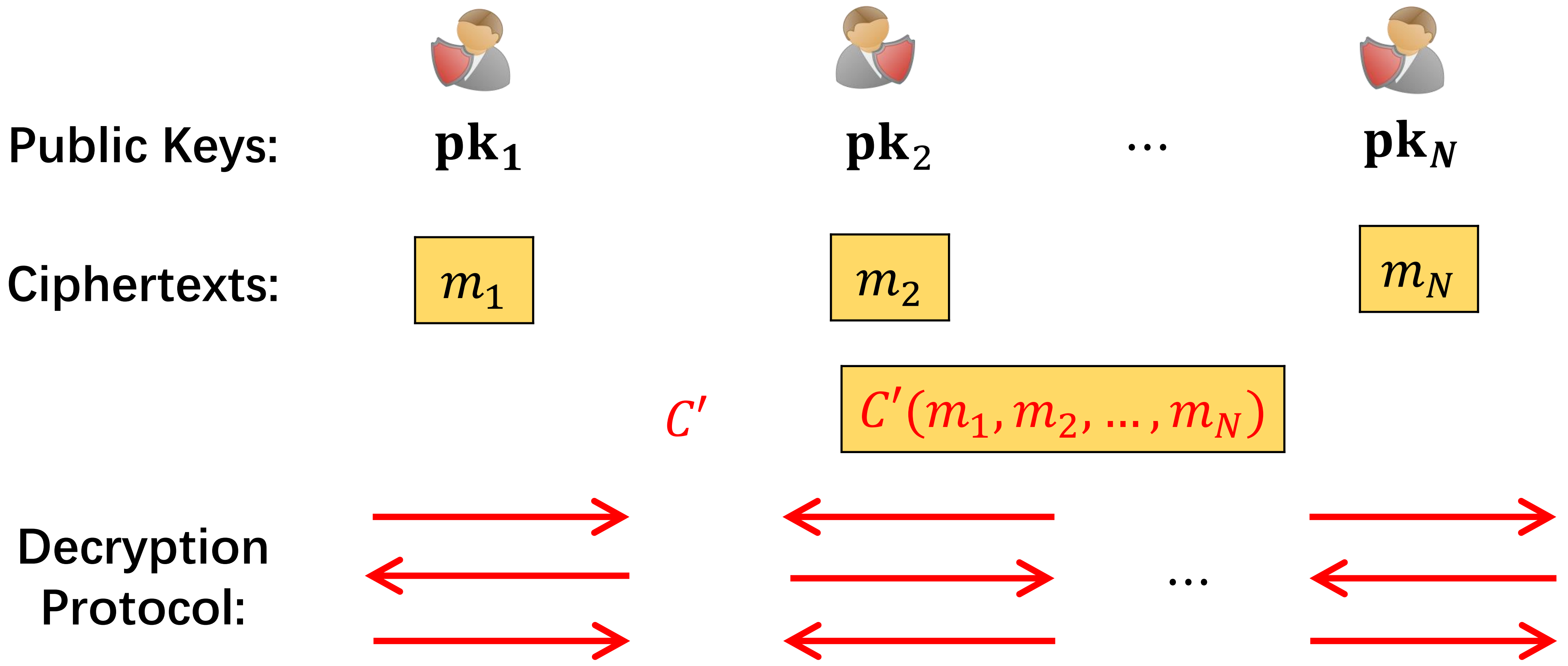

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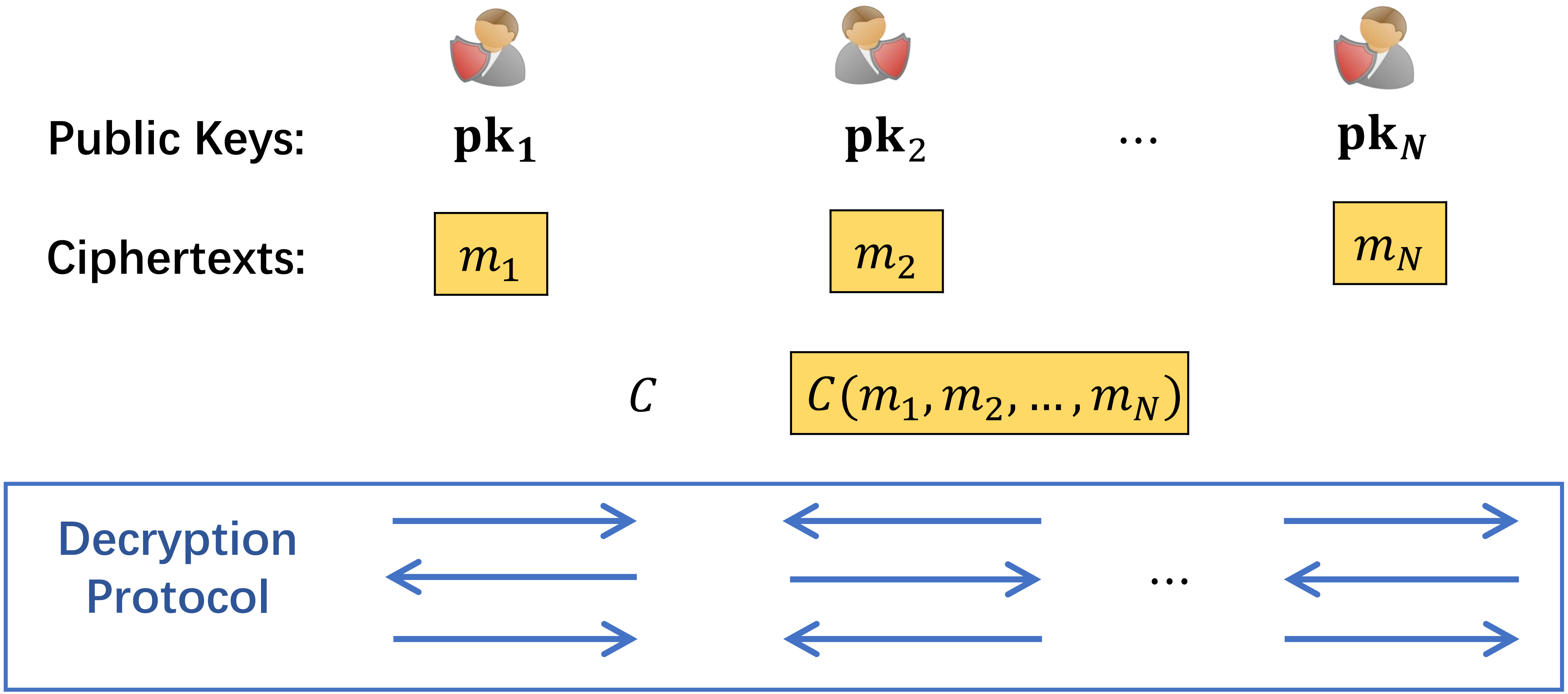
MK-FHE with 1-Round Decryption [MW16]



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1-round Decryption: → ← ⋯ →

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C $C(m_1, m_2, \dots, m_N)$

1-round Decryption: $\cdots$

Public Recovery: $C(m_1, m_2, \dots, m_N)$

Applications

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- 2-round multiparty computation [MW16]
- Spooky encryption [DHRW16]
- Homomorphic secret sharing [BGI16, BGI17]
- obfuscation & functional encryption combiners [AJNSY16, AJS17]
- Multiparty obfuscation [HIJKSY17]
- Homomorphic time-lock puzzles [MT19,BDGM19]
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Prior works on
Multi-key FHE with 1-round decryption

- [CM15, MW16, BP16, PS16] need a trusted setup.
- [DHRW16] sub-exponentially secure indistinguishable obfuscation.

In the plain model, does Multi-key FHE with 1-round decryption exist?

Our Results

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1. Multi-key FHE with 1-round decryption in the plain model from Learning with Error (LWE), Ring-LWE, and Decisional Small Polynomial Ratio problem.
   - O(1)-party Multi-key FHE from only LWE.

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2. Multiparty Homomorphic Encryption (a weaker notion of MK-FHE) from LWE.

Multiparty Homomorphic Encryption: A weakening of MK-FHE



Public Keys: $\mathbf{pk}_1$ $\mathbf{pk}_2$ $\cdots$ $\mathbf{pk}_N$

Ciphertexts: m_1 m_2 m_N

C $C(m_1, m_2, \ldots, m_N)$

Partial Decryption: $\longrightarrow$ $\longleftarrow$ $\cdots$ $\longrightarrow$

Public Recovery: **Partial Decryptions** $\rightarrow C(m_1, m_2, \ldots, m_N)$

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Partial Decryption: $\longrightarrow$ $\longleftarrow$ $\cdots$ $\longrightarrow$

Public Recovery: C, **Partial Decryptions** $\rightarrow C(m_1, m_2, \ldots, m_N)$

- It implies 2-round reusable multiparty computation with compact communication complexity.

Multiparty Homomorphic Encryption: A weakening of MK-FHE

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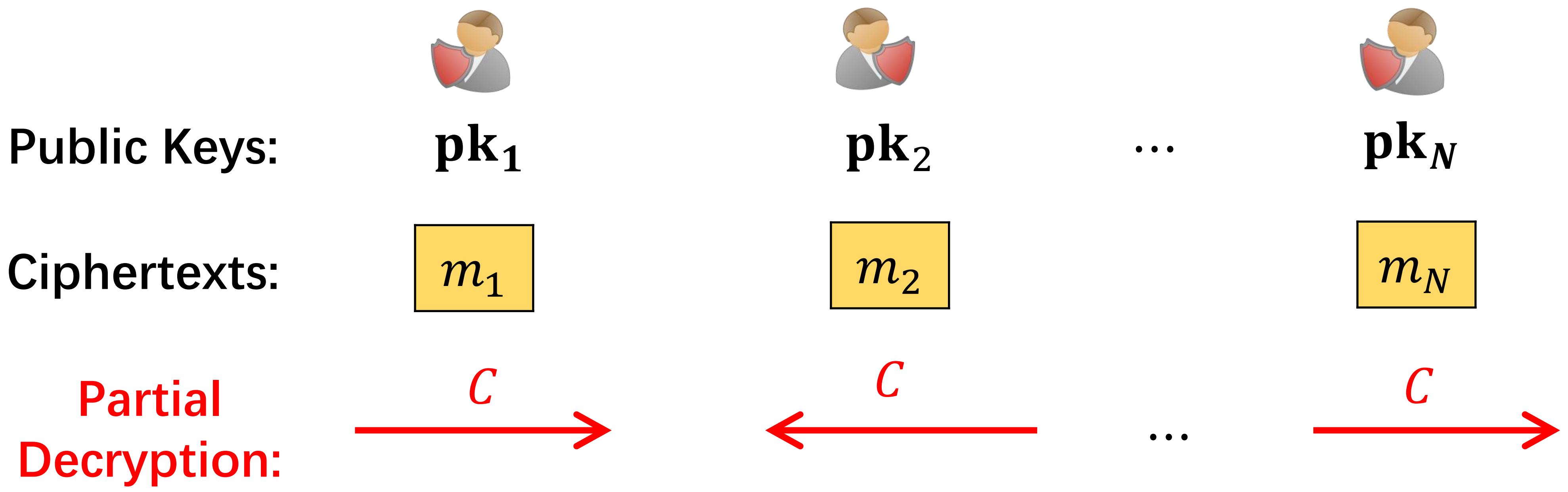

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Public Recovery: C, **Partial Decryptions** $\rightarrow C(m_1, m_2, \ldots, m_N)$

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- **Reusability:** public keys can be reused for different circuits.
- **Compactness:** communication complexity is independent of the circuit.

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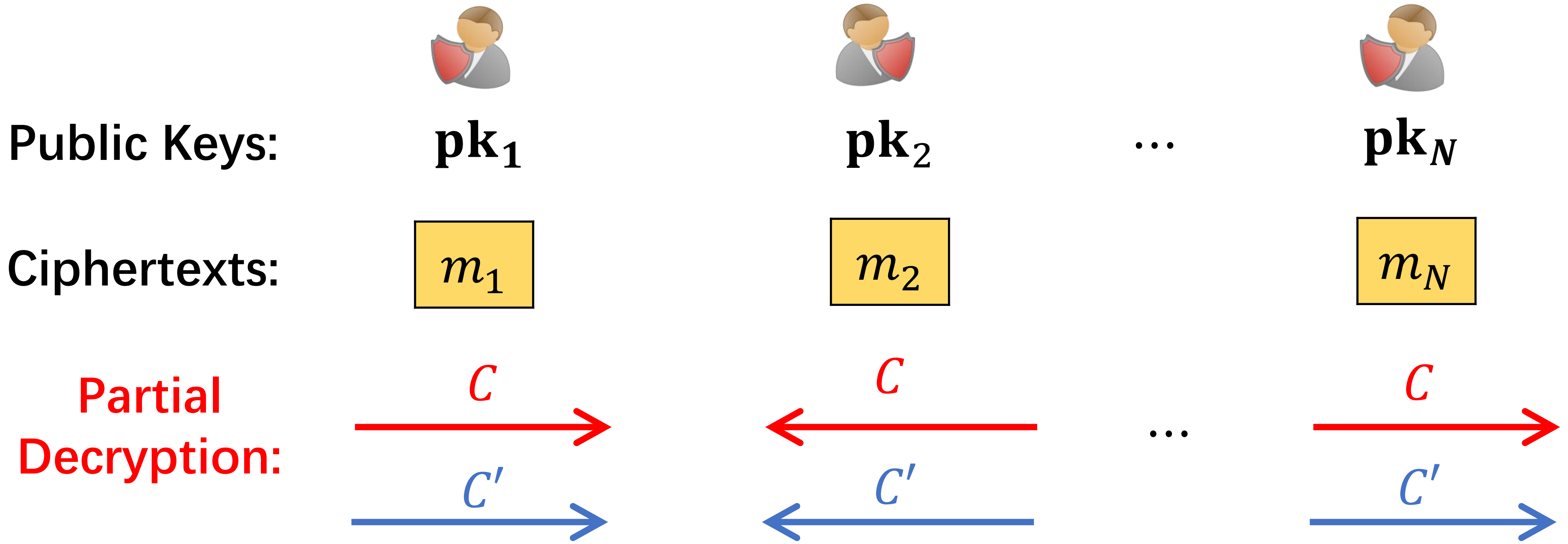

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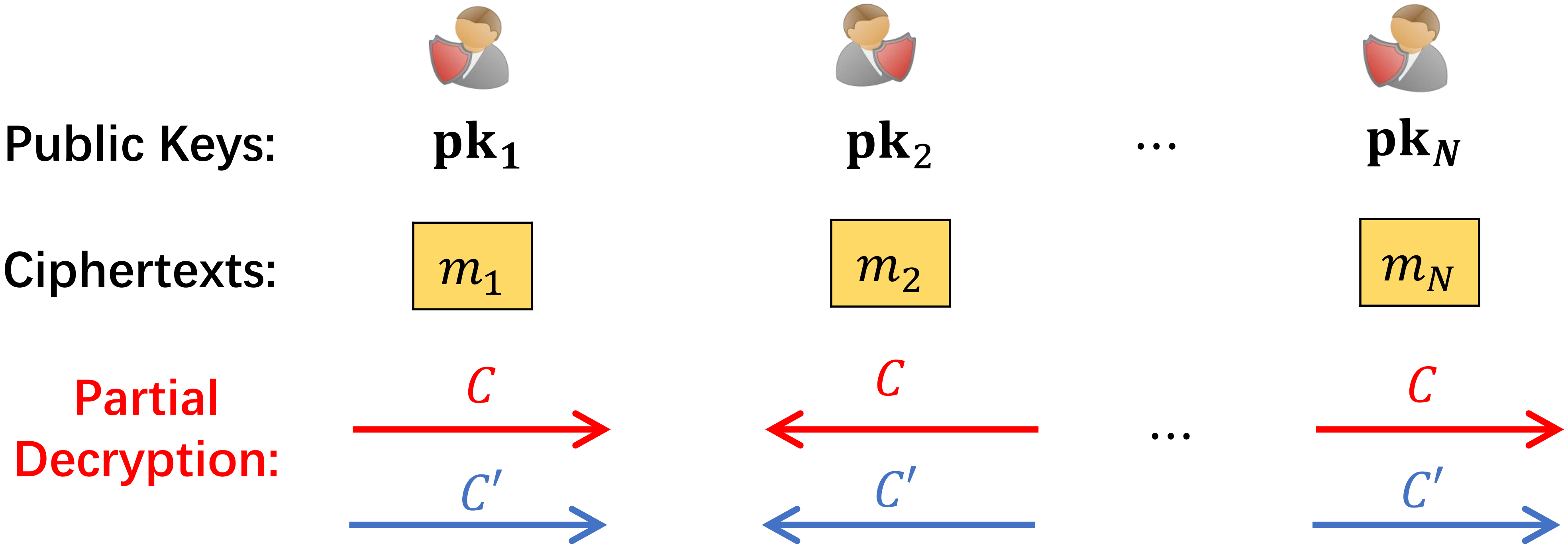

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- It implies 2-round Multiparty Computation.

Our Approach

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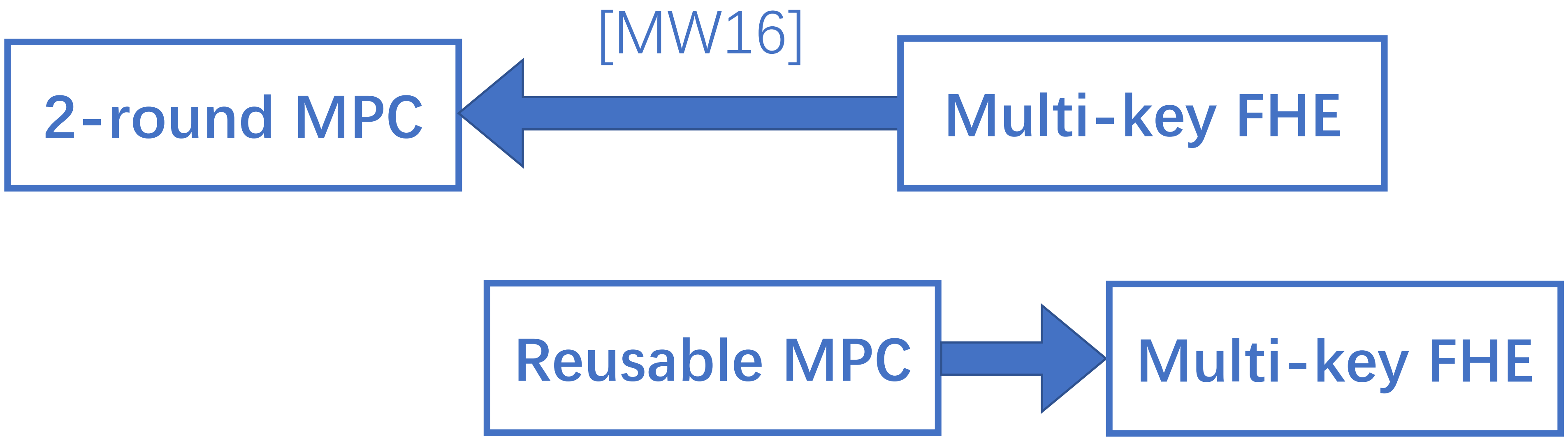

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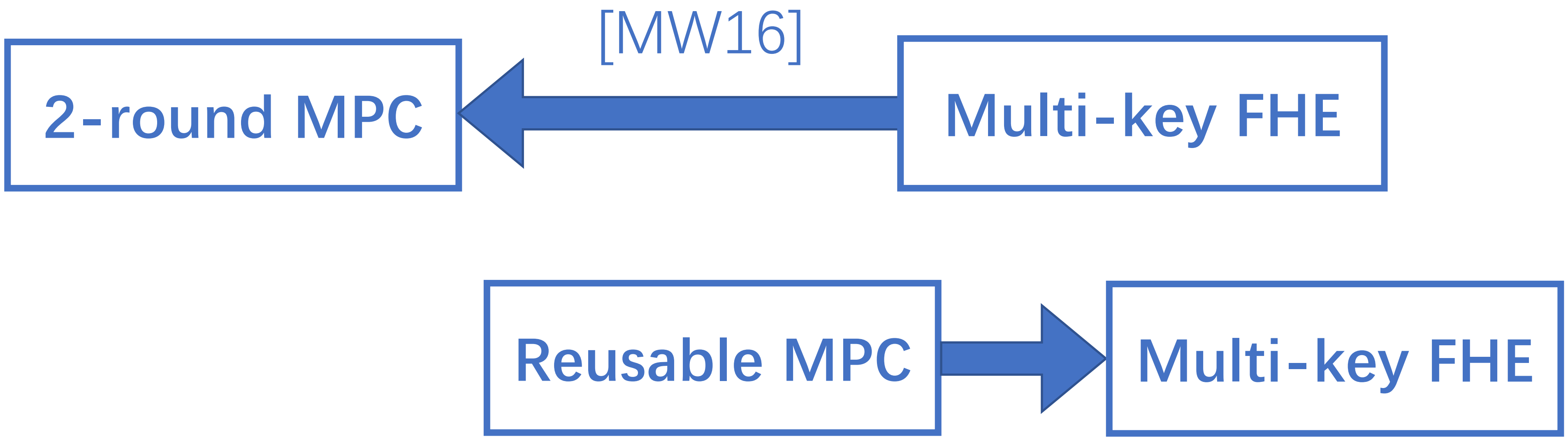

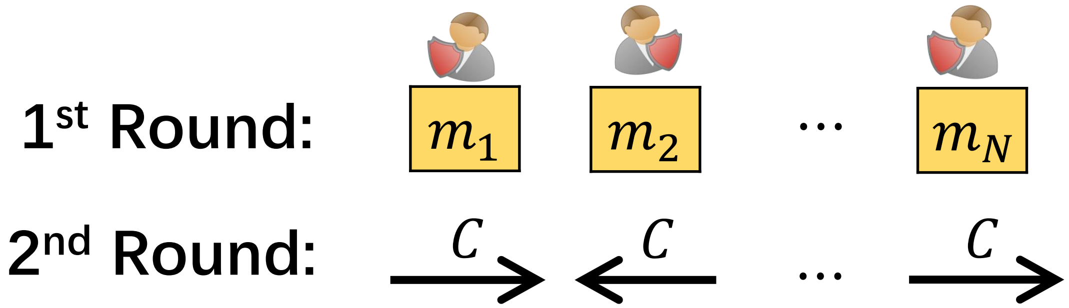

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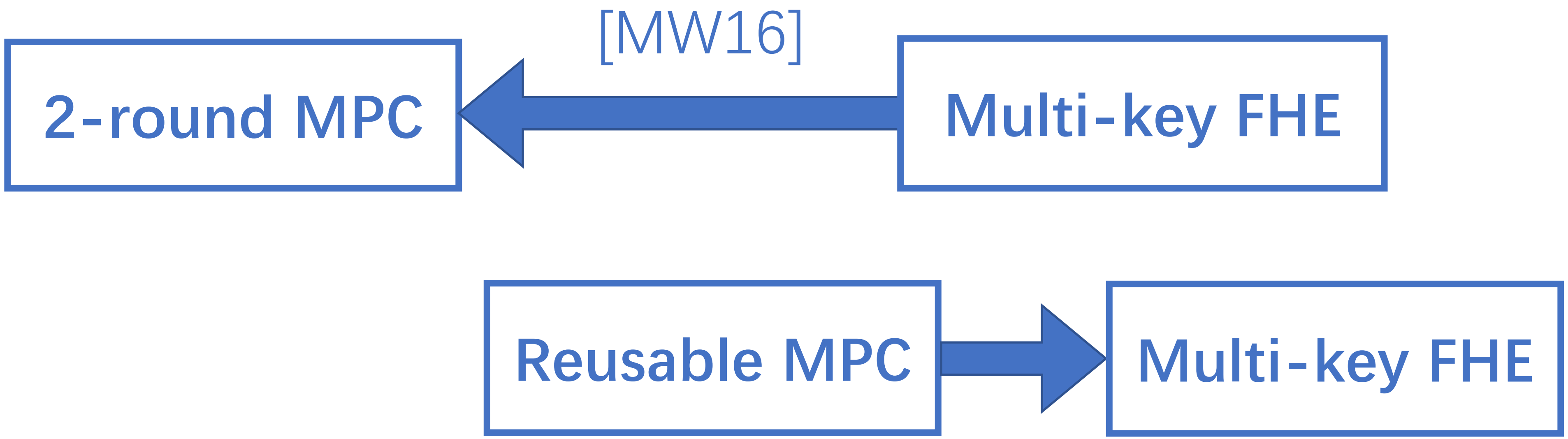

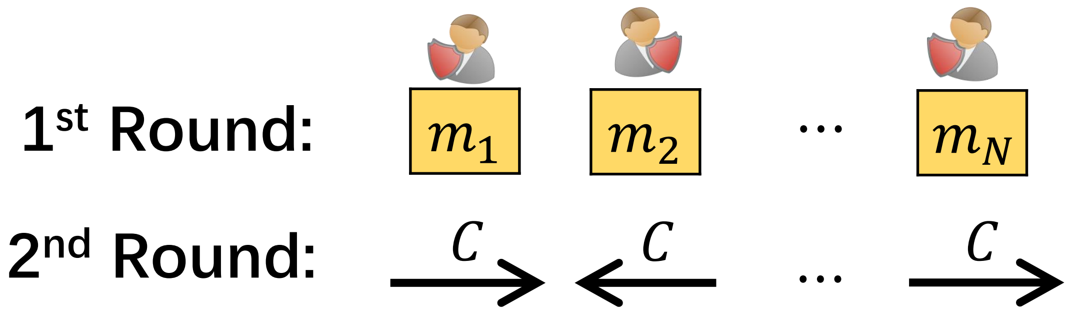

- **Reusability**: 1st round is reusable.
- **See Also**:
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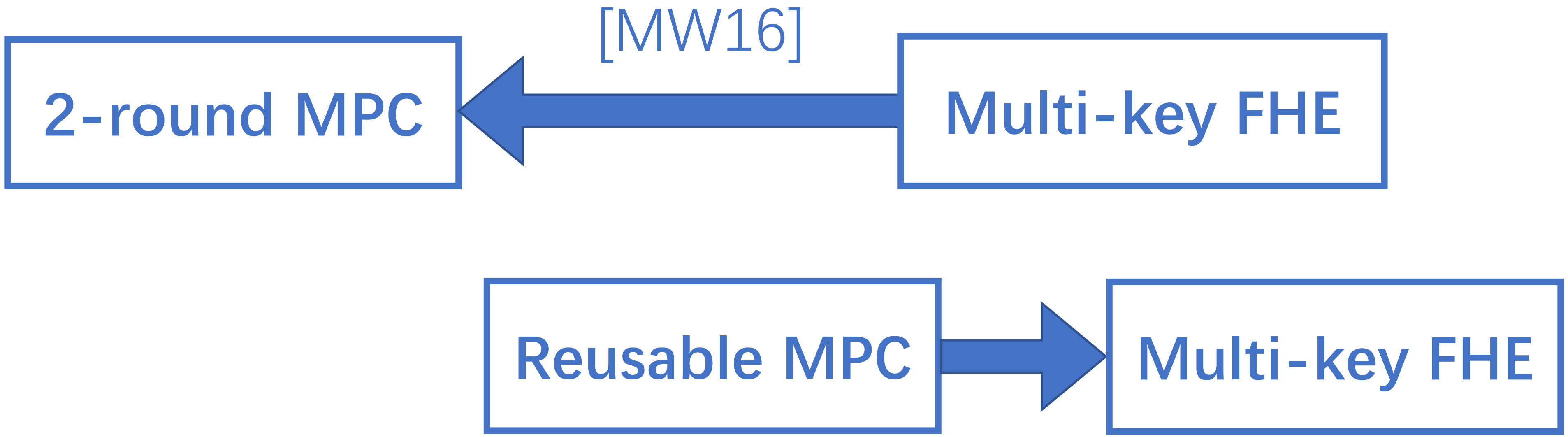

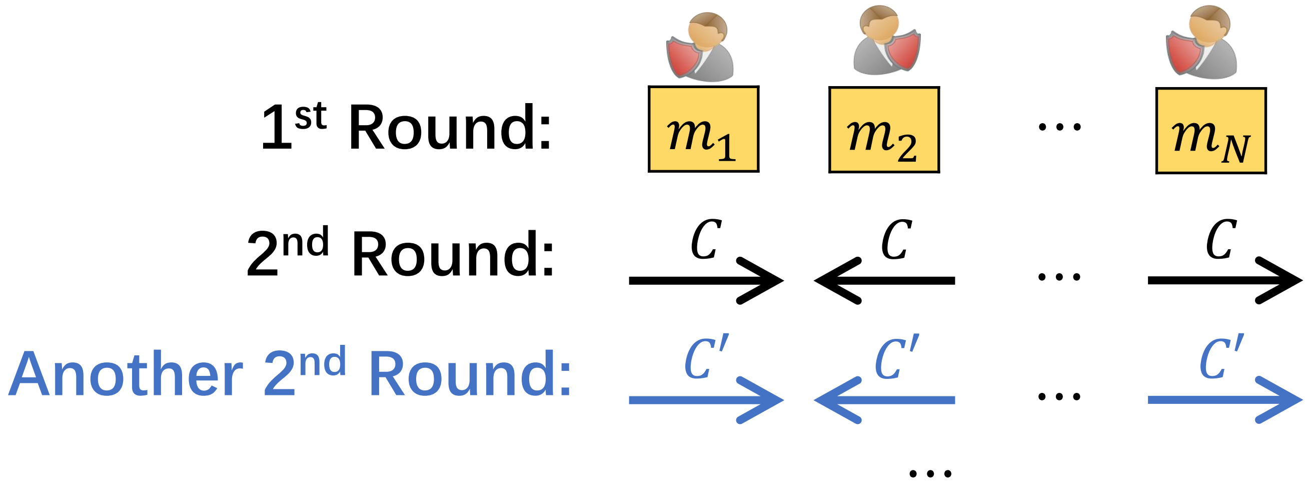

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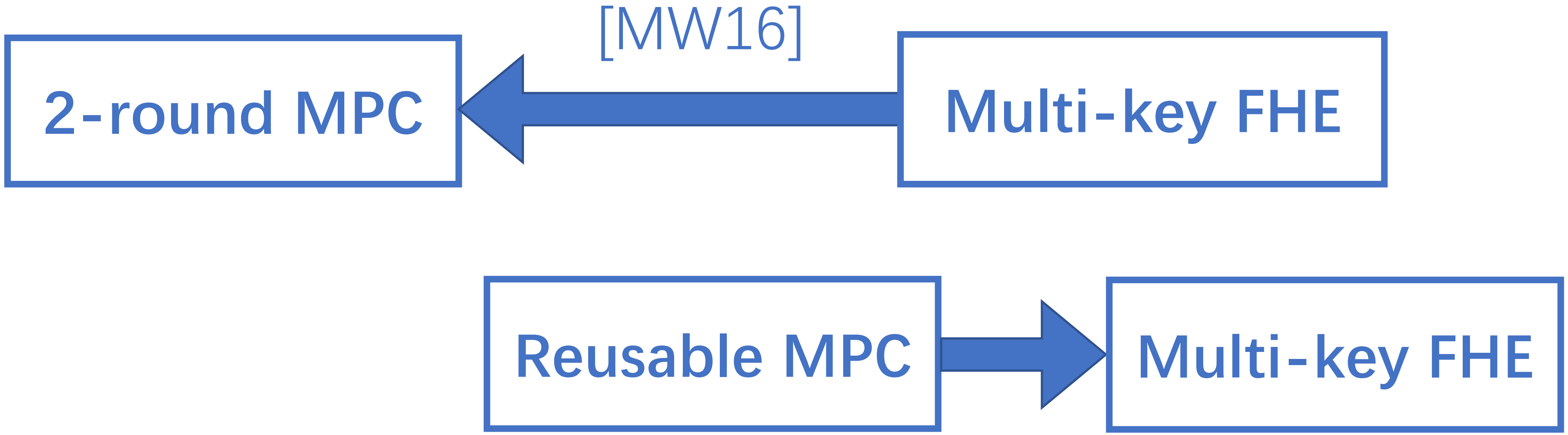

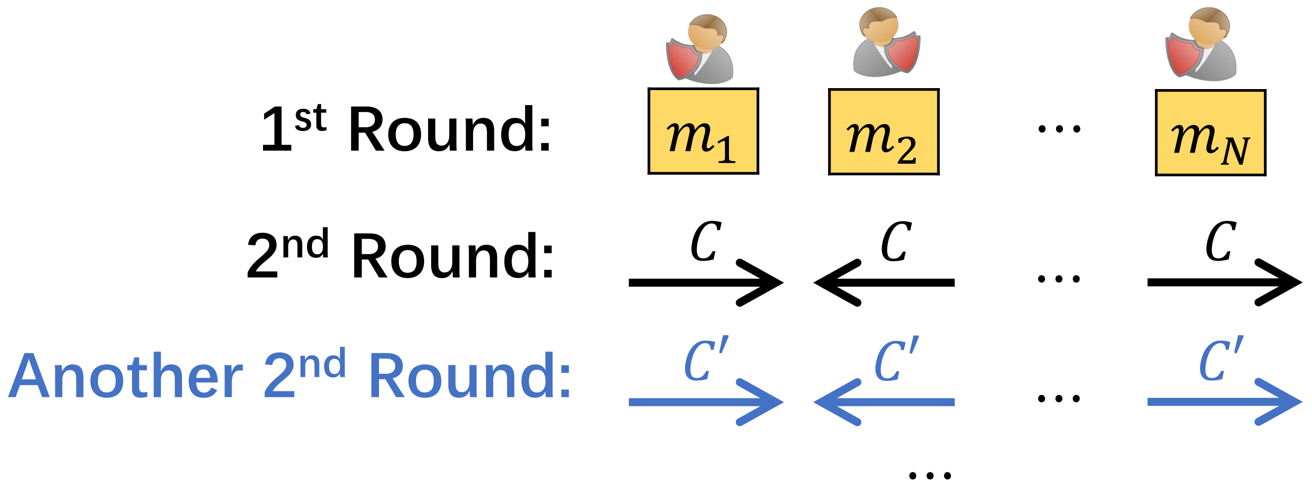

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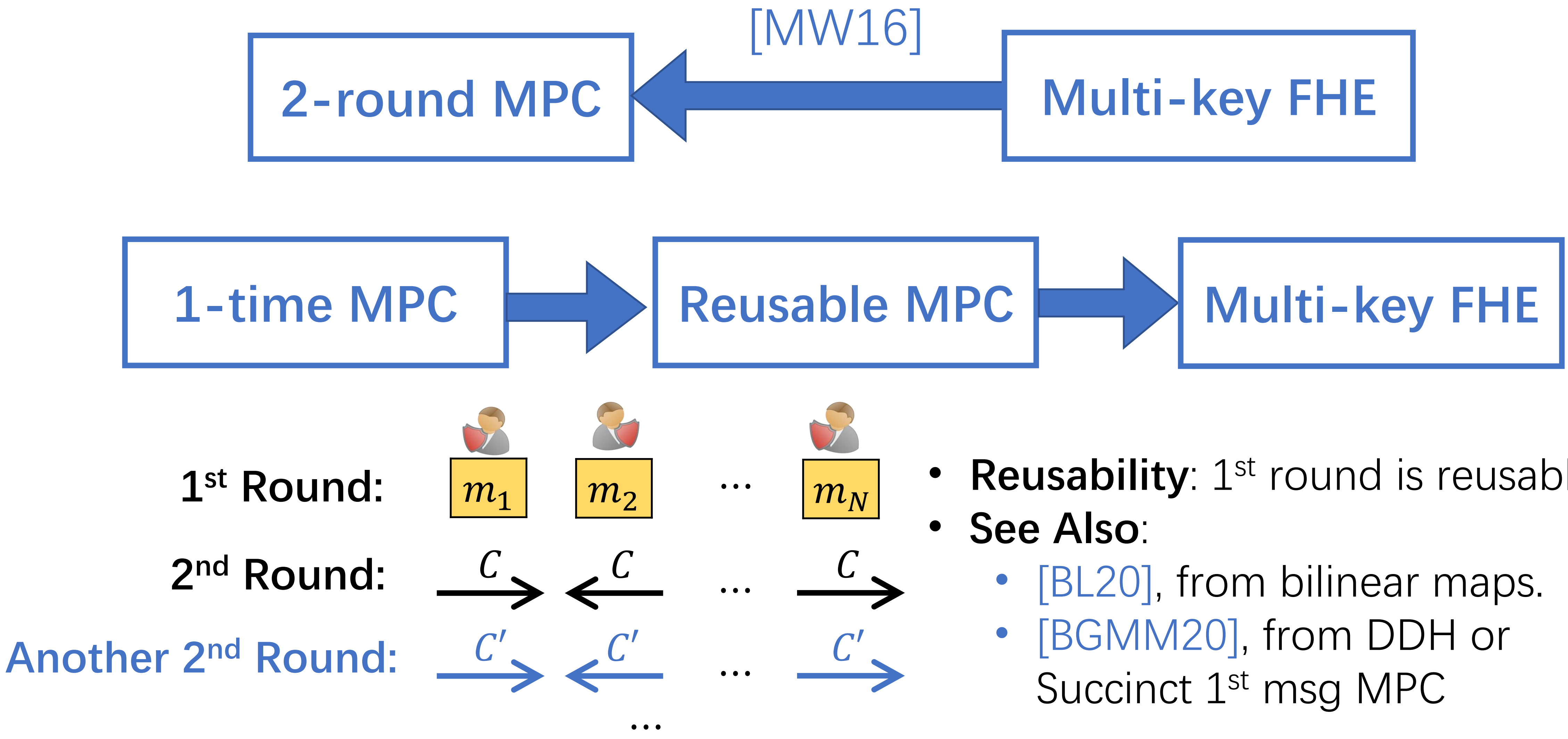

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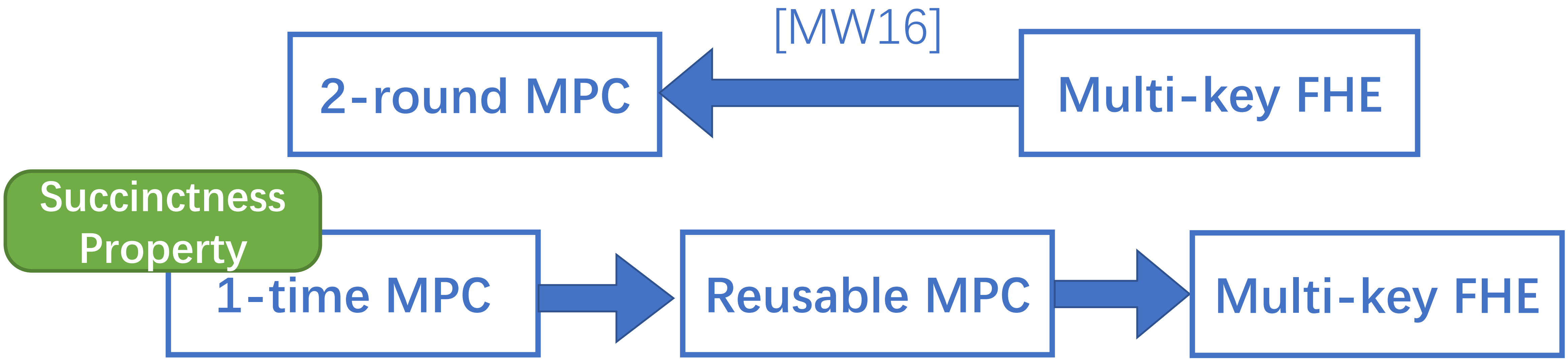

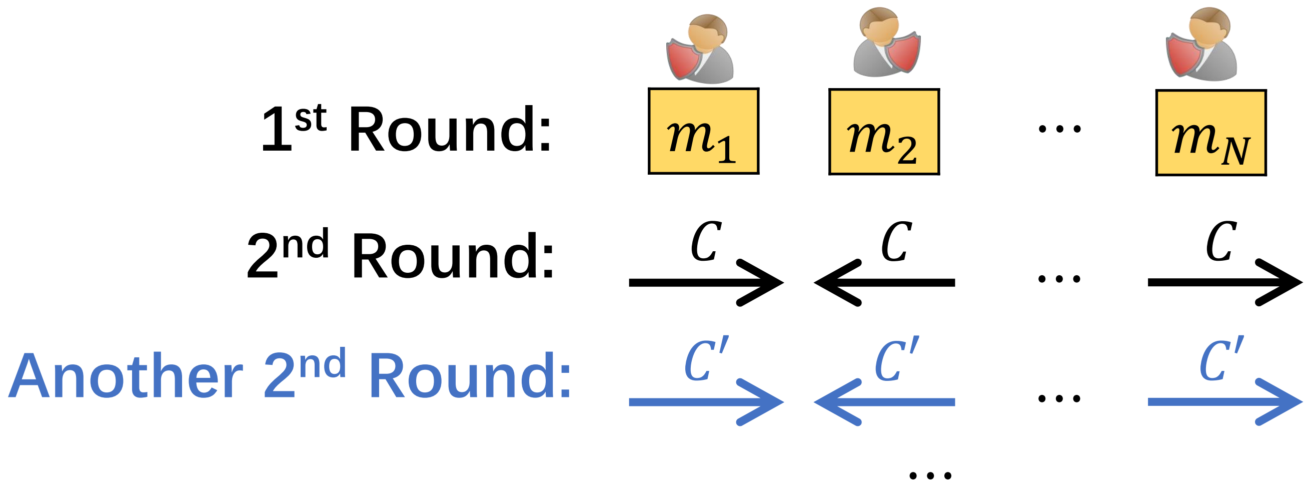

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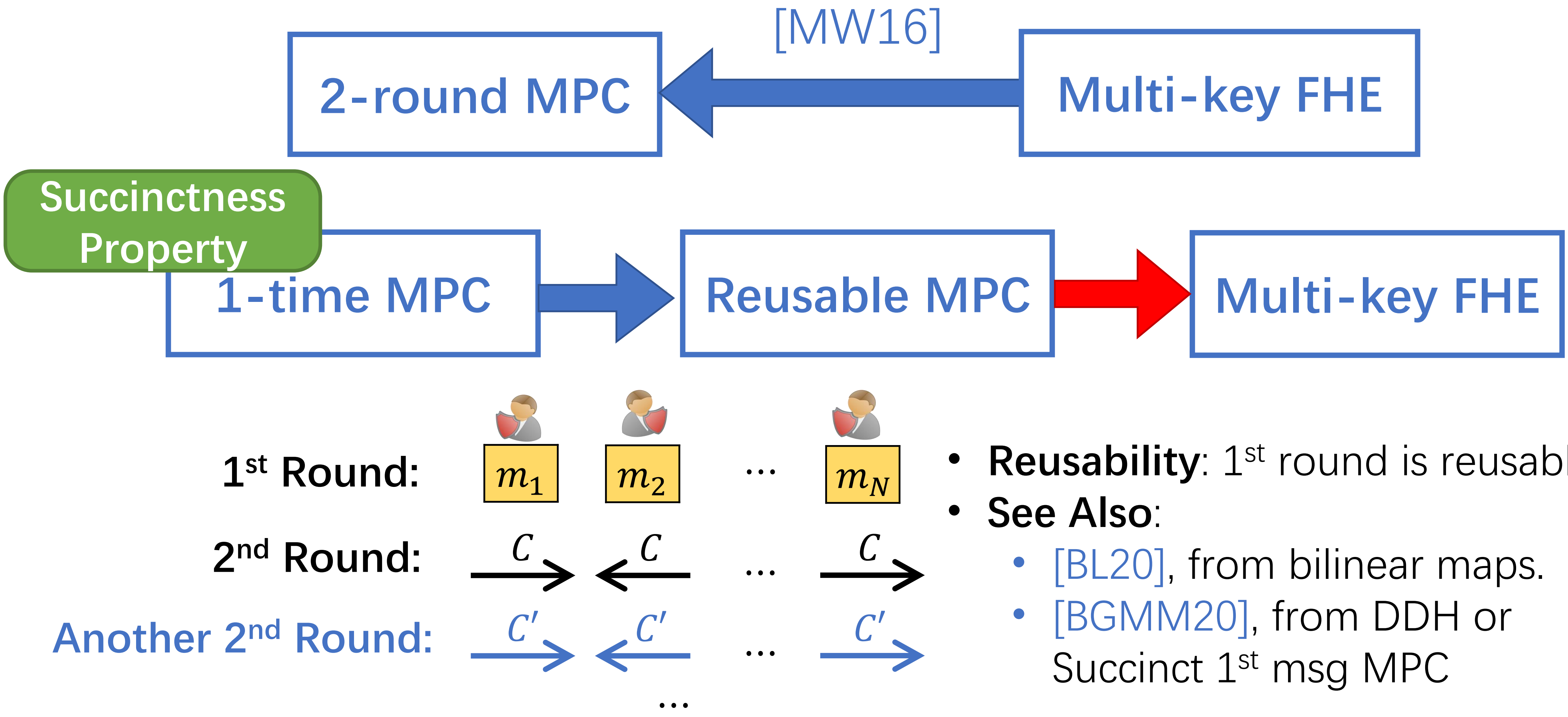

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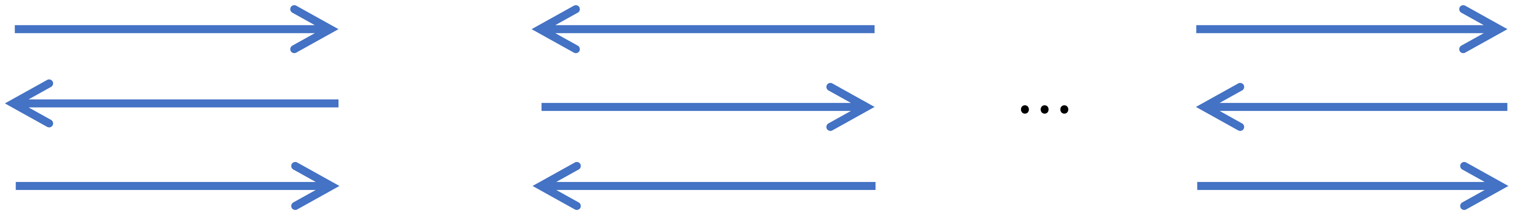
Multi-round Decryption:



$C(m_1, m_2, \ldots, m_N)$

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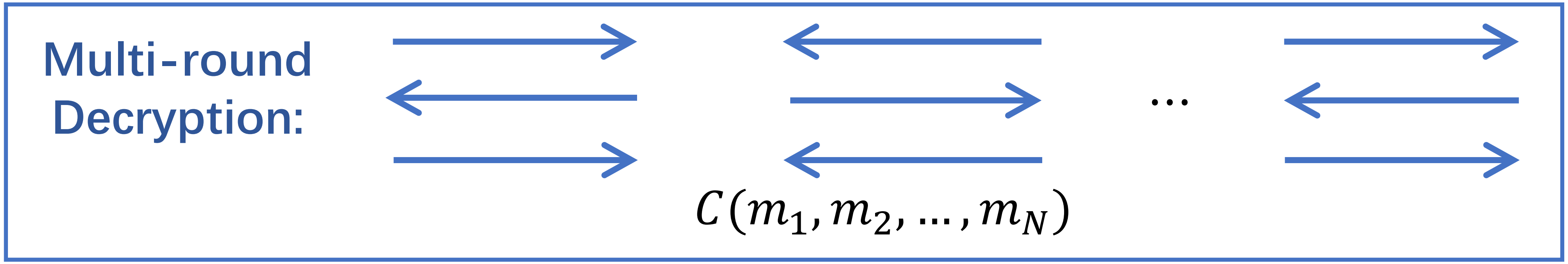
- Run reusable MPC for $\mathbf{Dec}(\cdot)$.



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Public Keys & Reusable MPC 1st Round

$\mathbf{pk}_1$ sk_1

$\mathbf{pk}_2$ sk_1 …

$\mathbf{pk}_N$ sk_N

Ciphertexts: m_1 m_2 m_N

C $C(m_1, m_2, \dots, m_N)$

- Run reusable MPC for $\mathrm{Dec}(\cdot)$.

Reusable MPC 2nd Round → ← … →

$C(m_1, m_2, \dots, m_N)$

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Reusable MPC: A Self-Synthesis Approach

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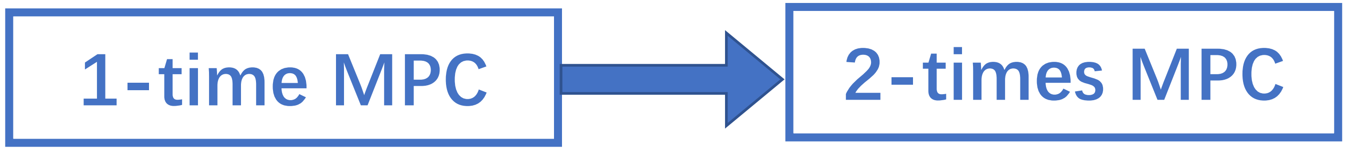
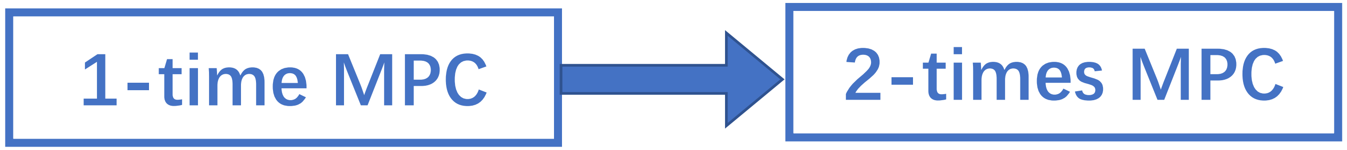


Reusable MPC: A Self-Synthesis Approach



- Use 1-time MPC to generate 2 sets of fresh new 1st round messages

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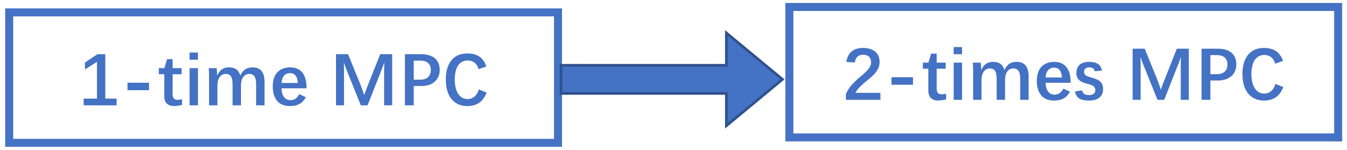

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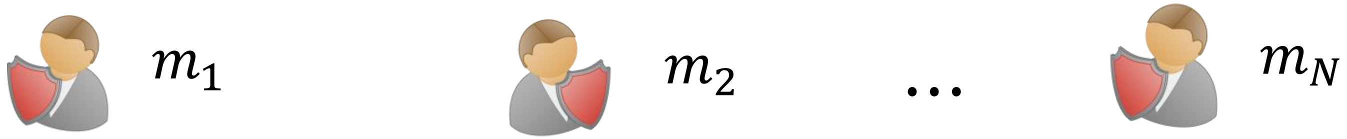
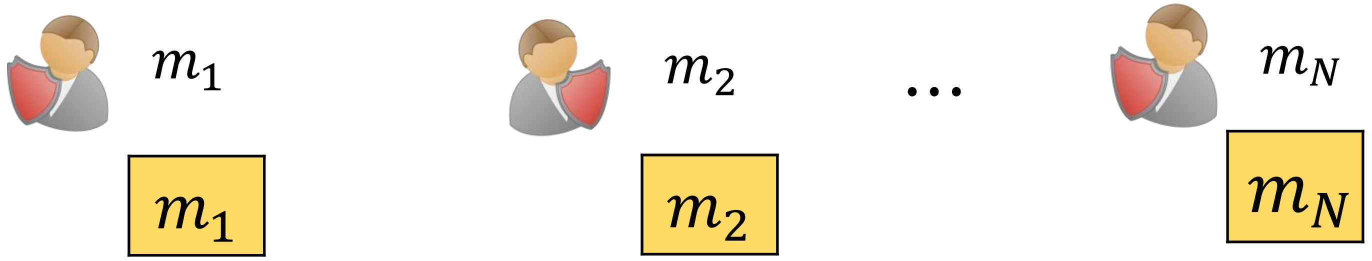


Reusable MPC: A Self-Synthesis Approach

1-time MPC → **2-times MPC**

- Use 1-time MPC to generate 2 sets of fresh new 1st round messages

1st Round:



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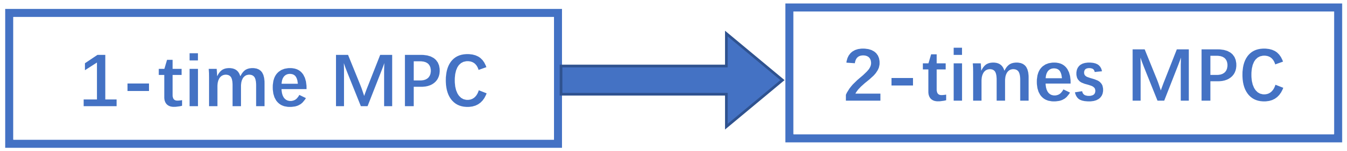

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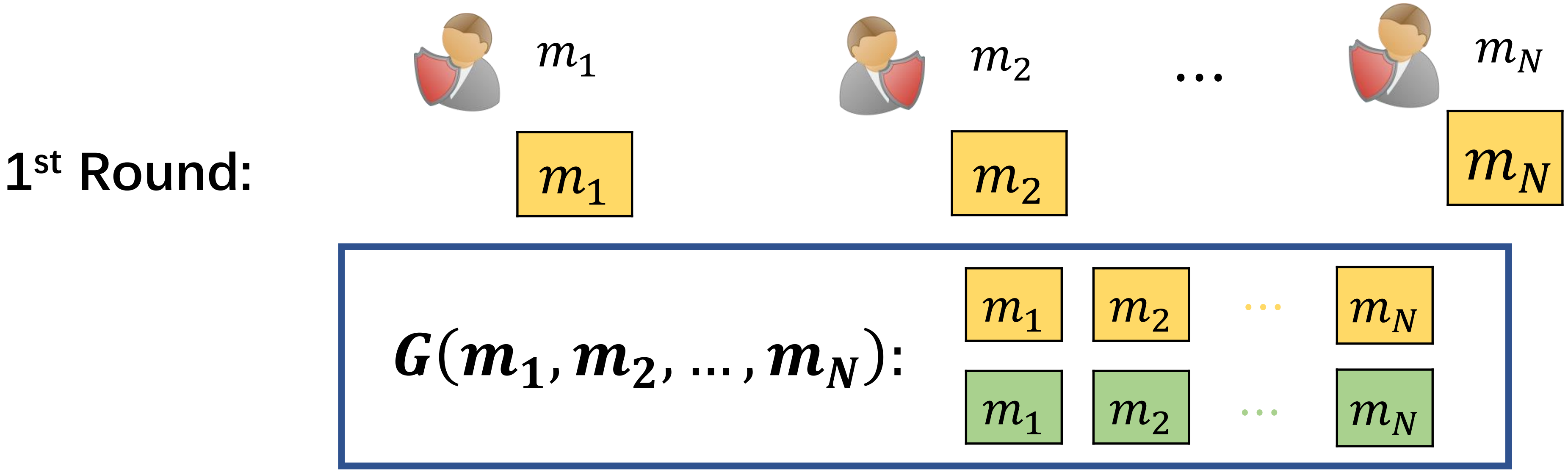

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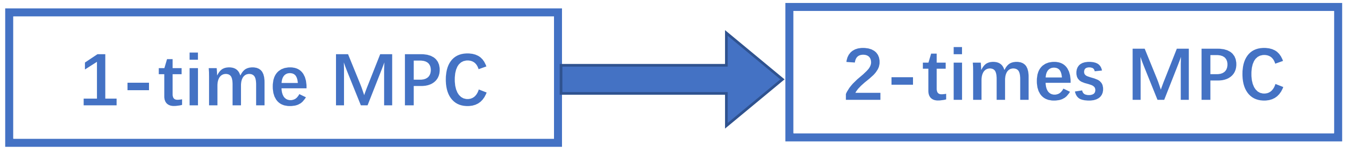

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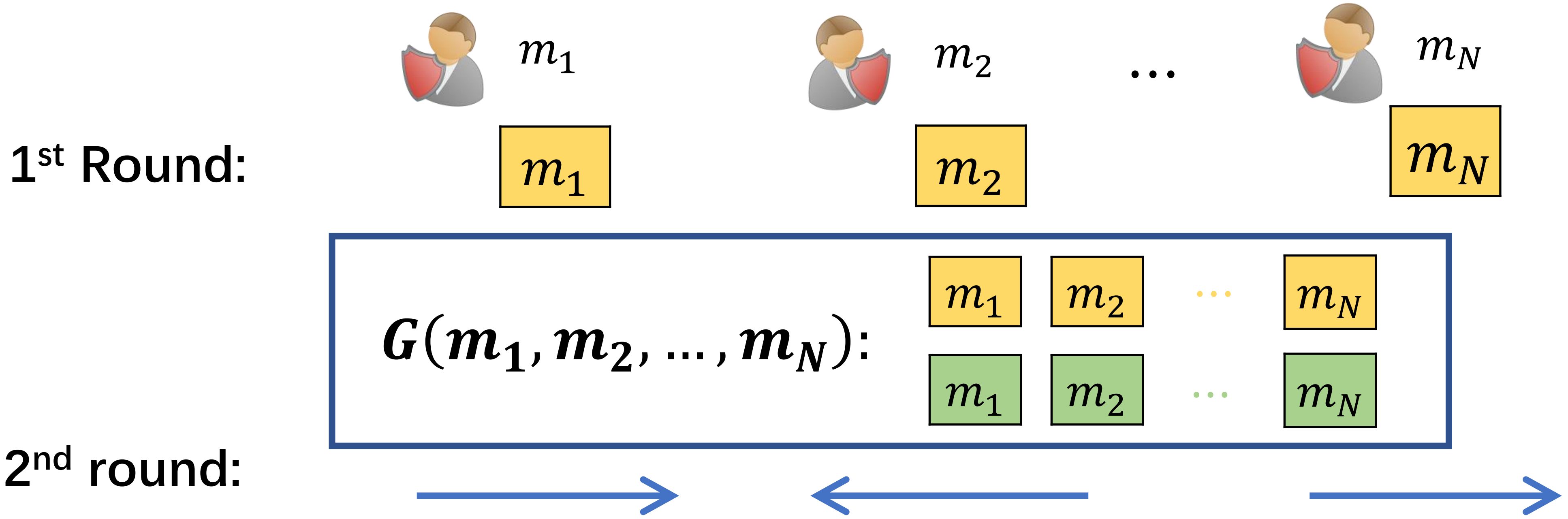

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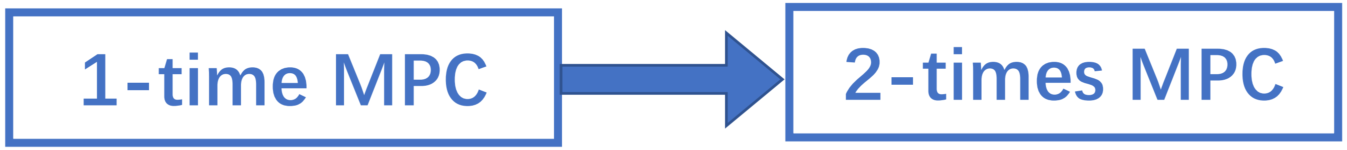

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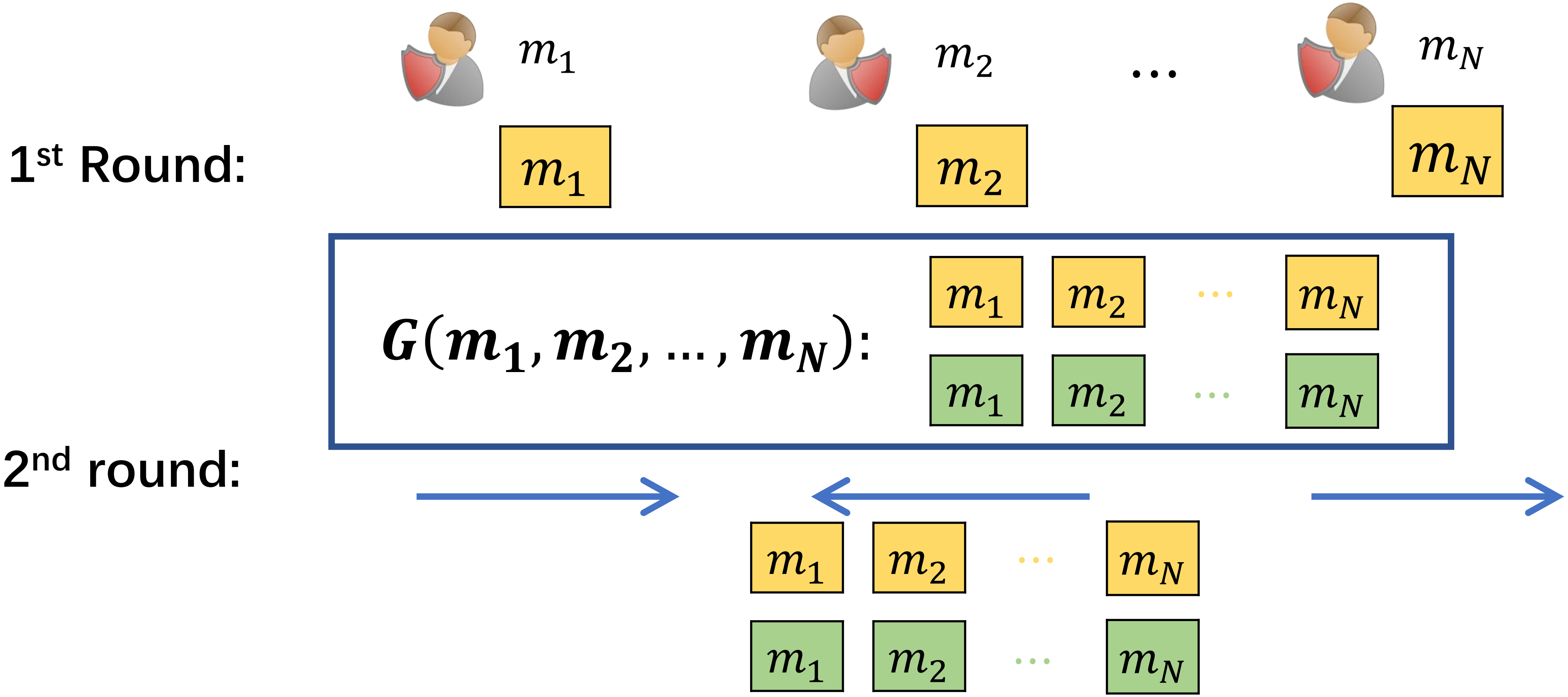

Reusable MPC: A Self-Synthesis Approach

1-time MPC → **2-times MPC**

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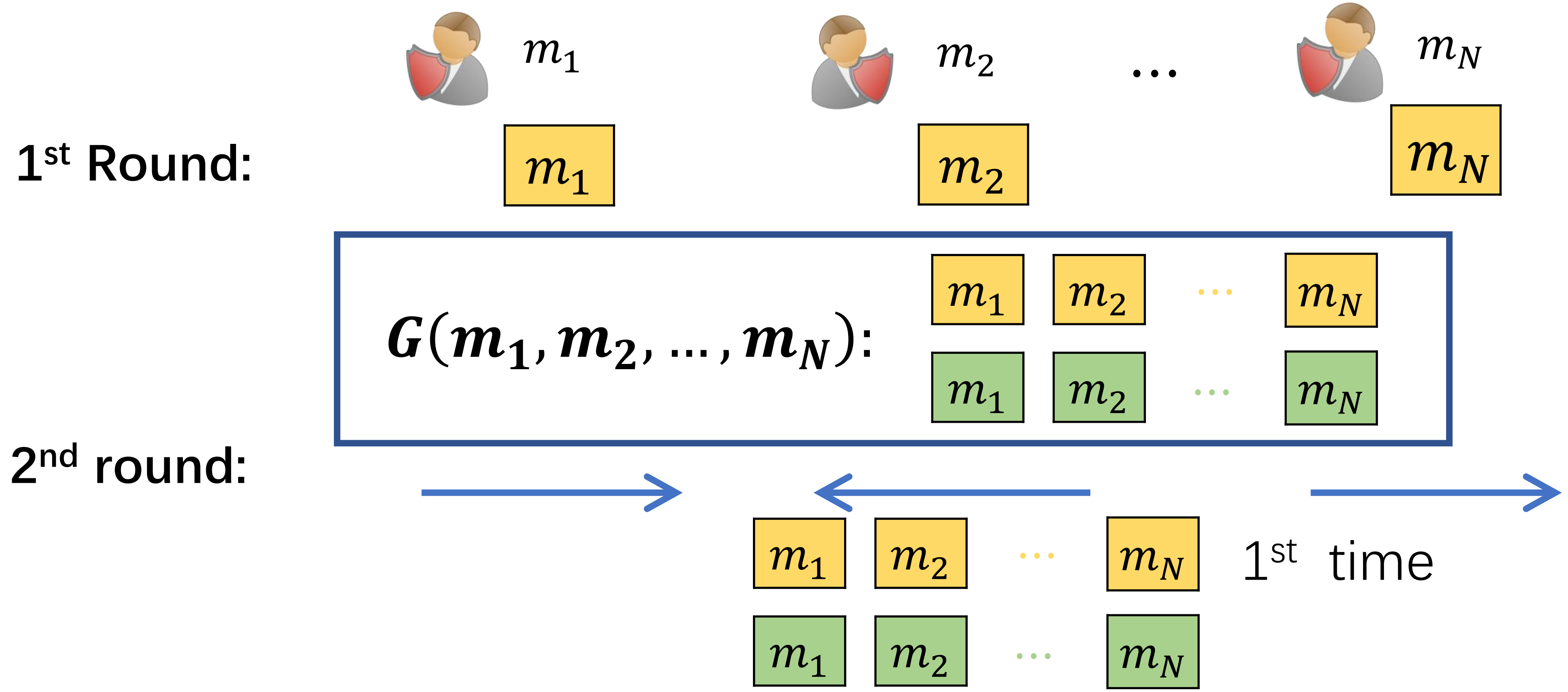

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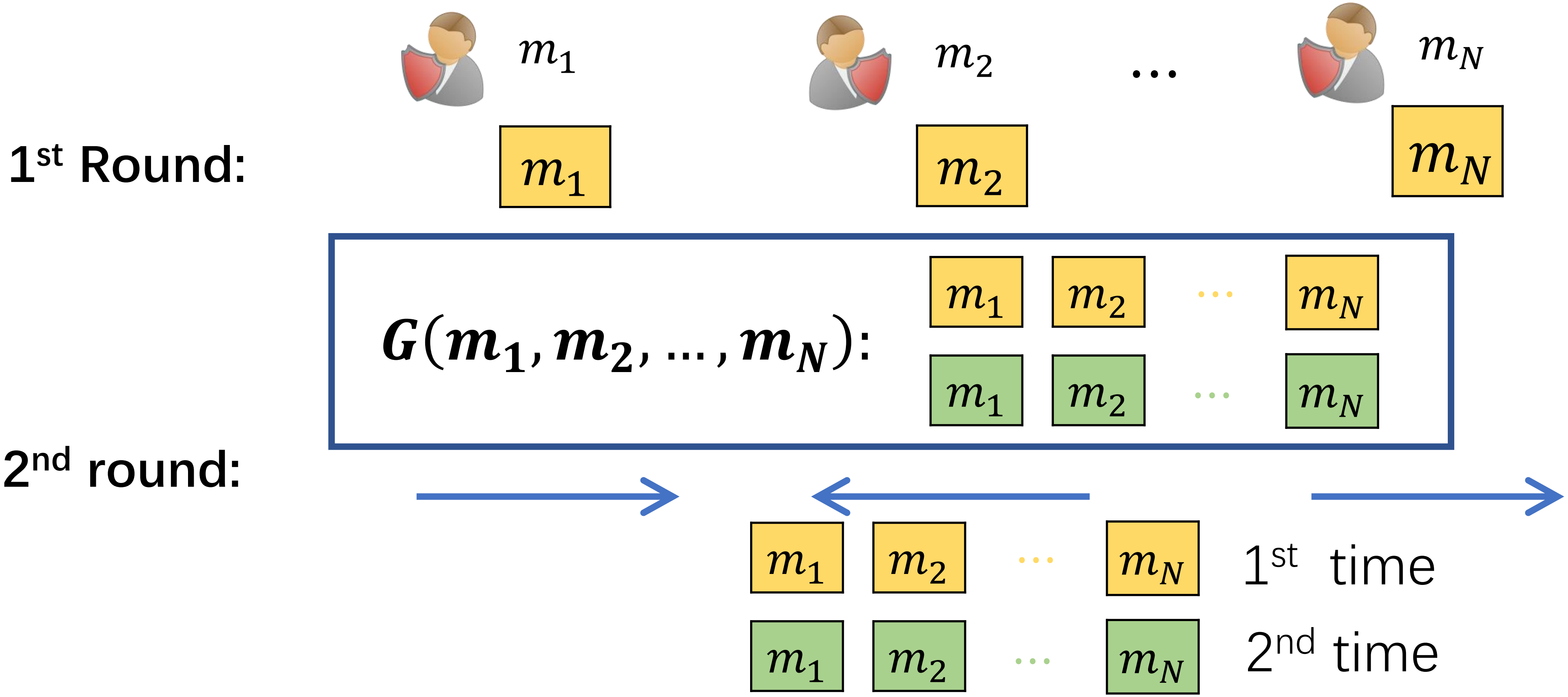

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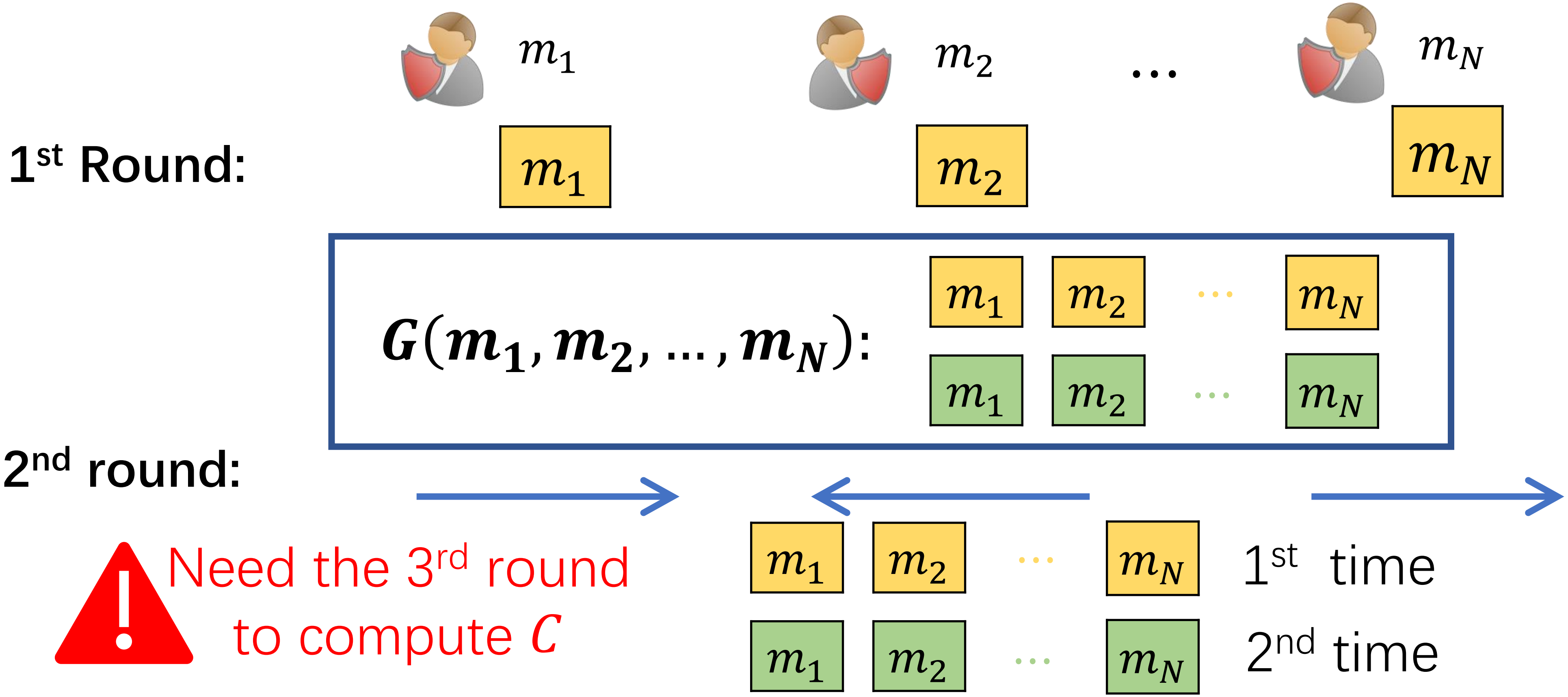

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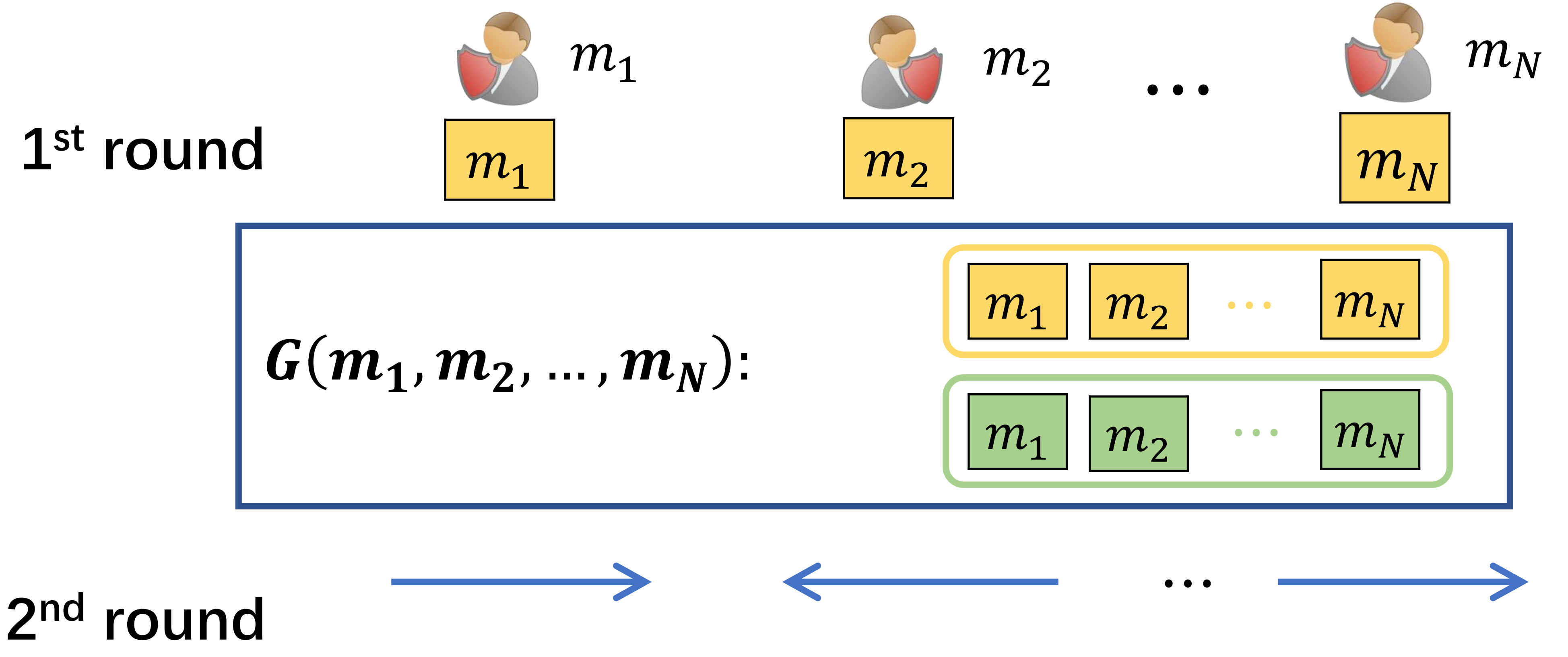

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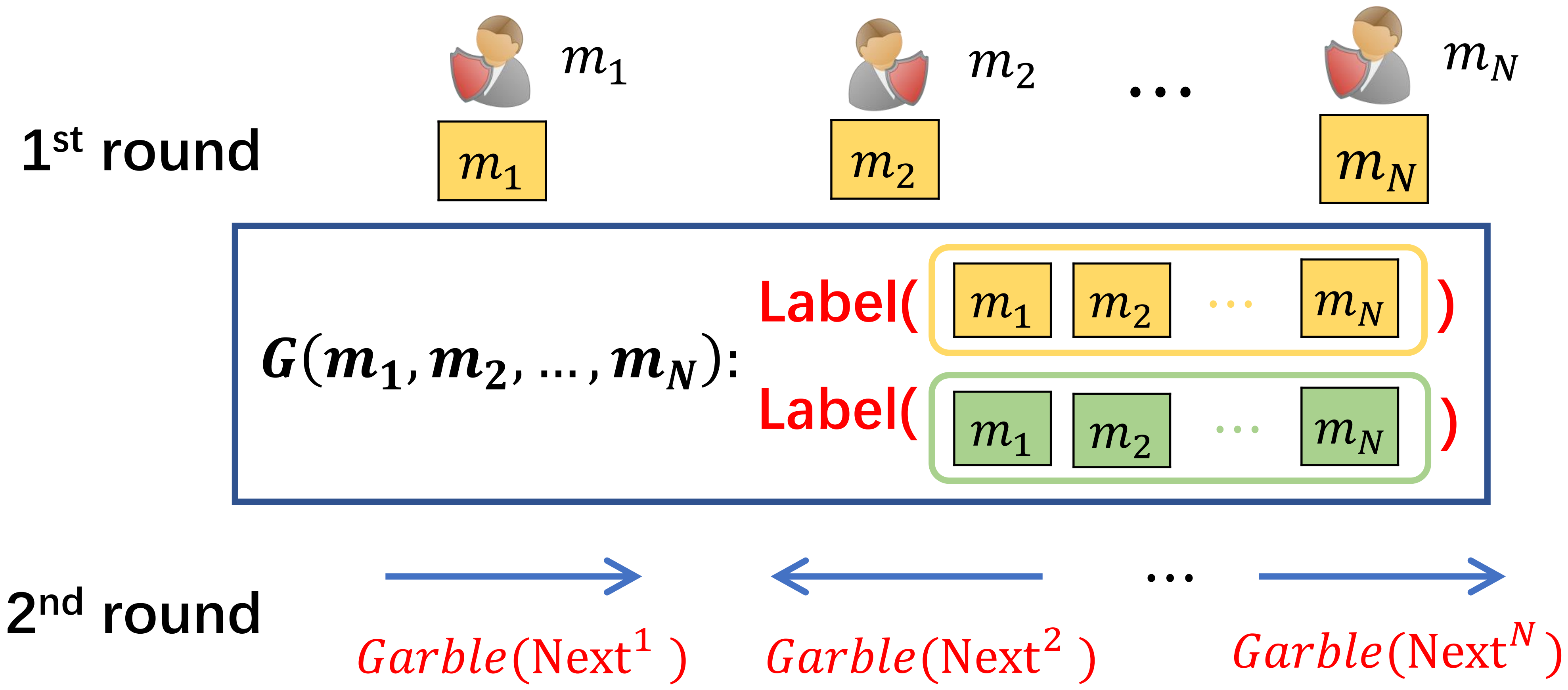

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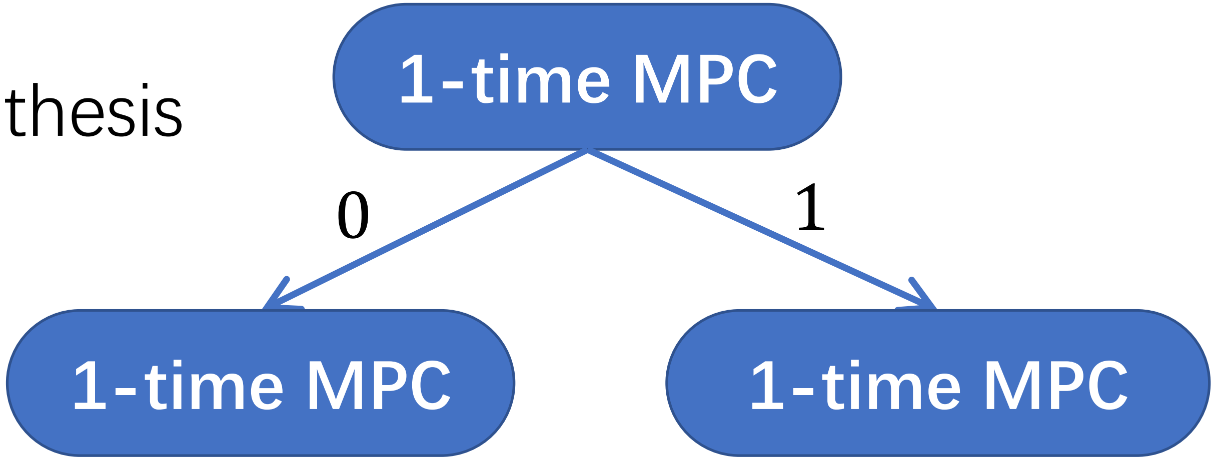

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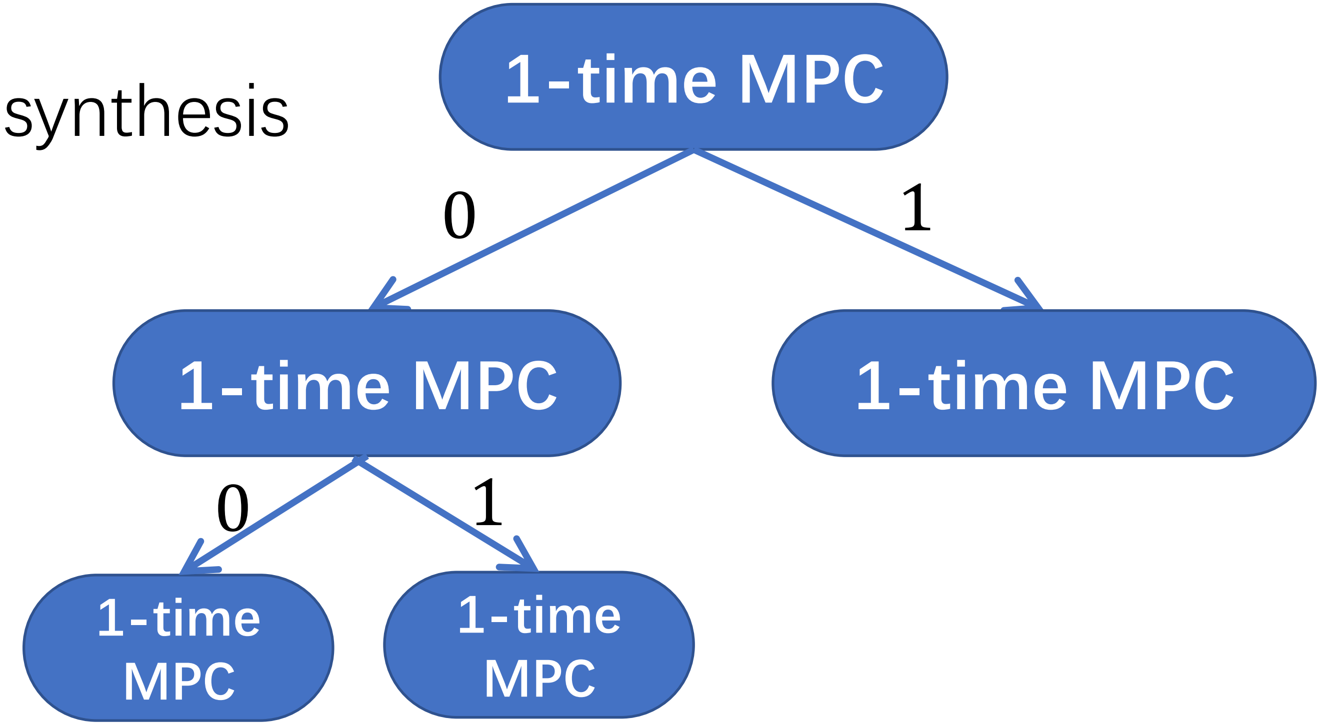

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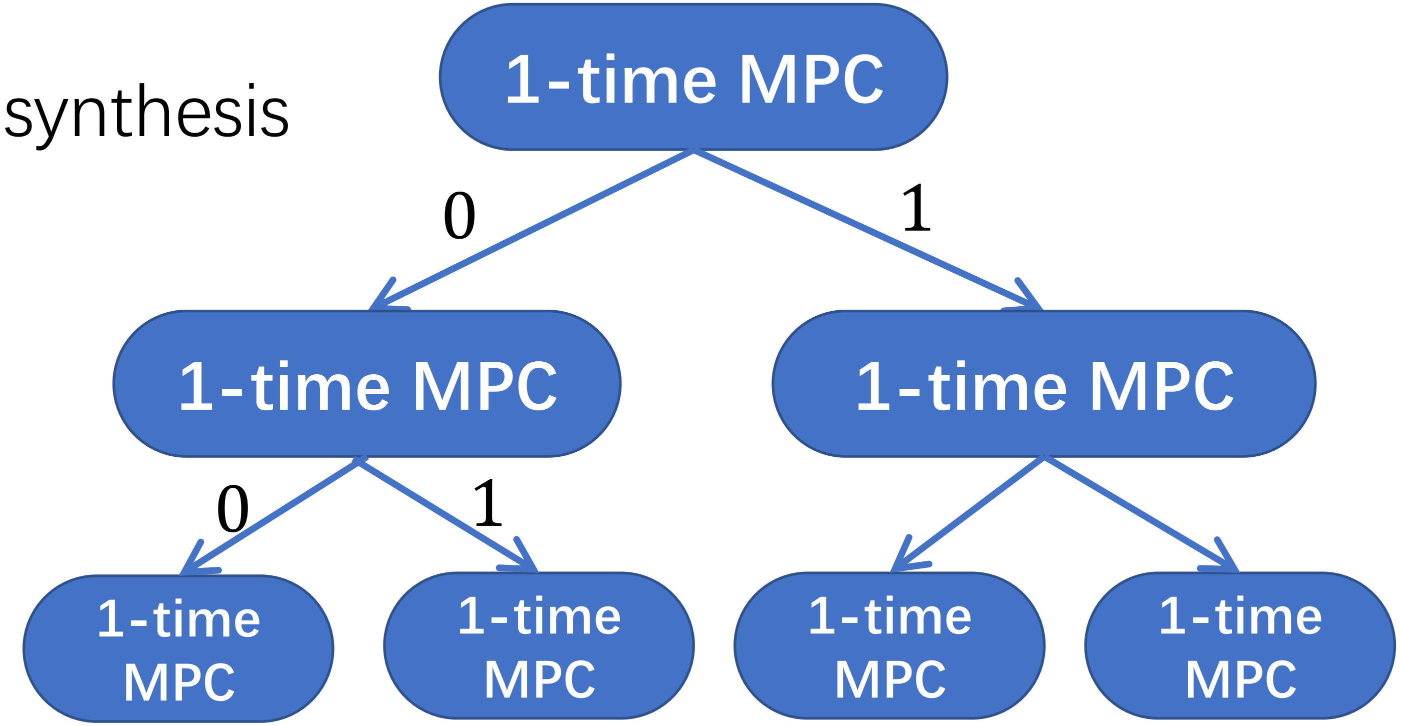

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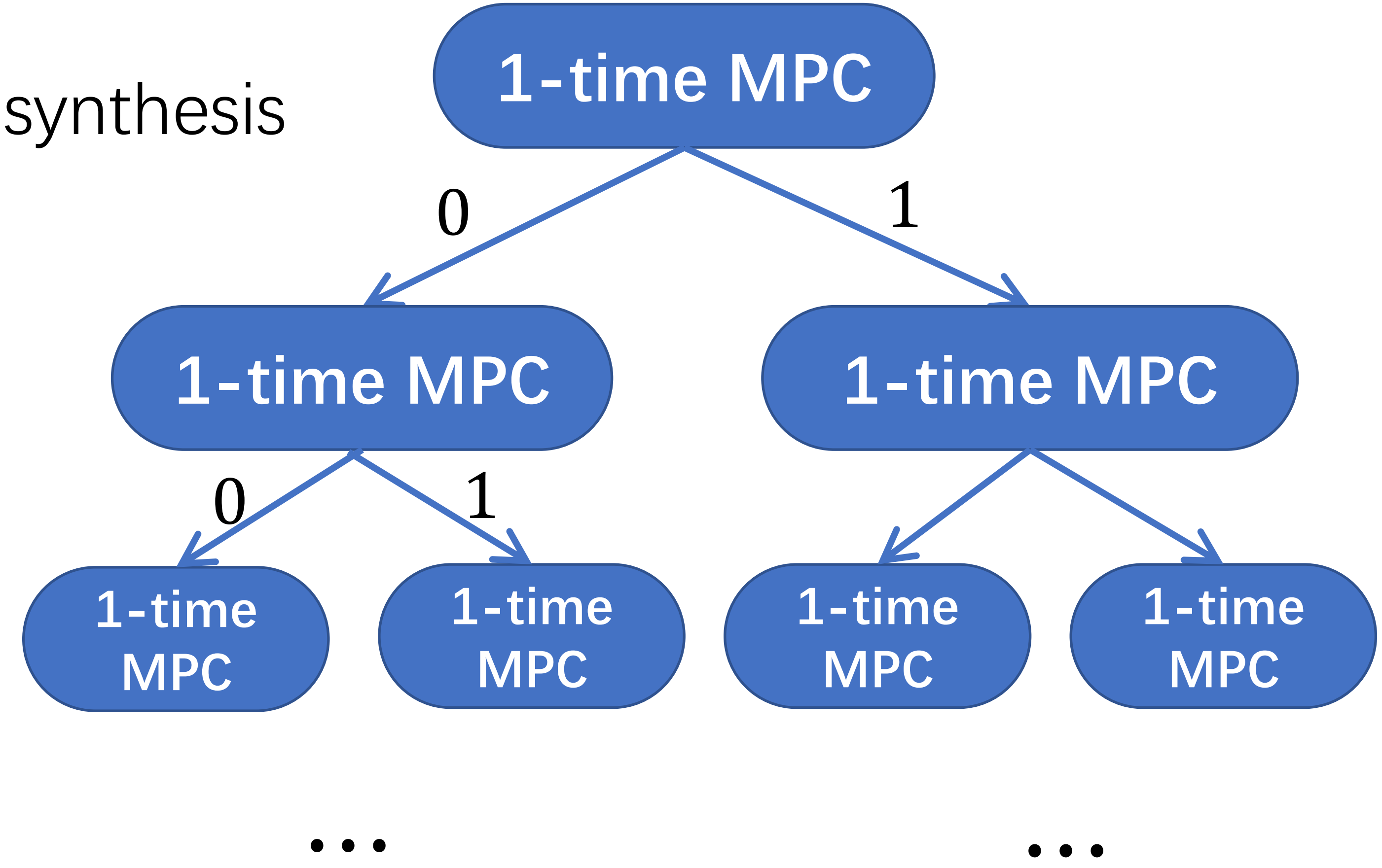

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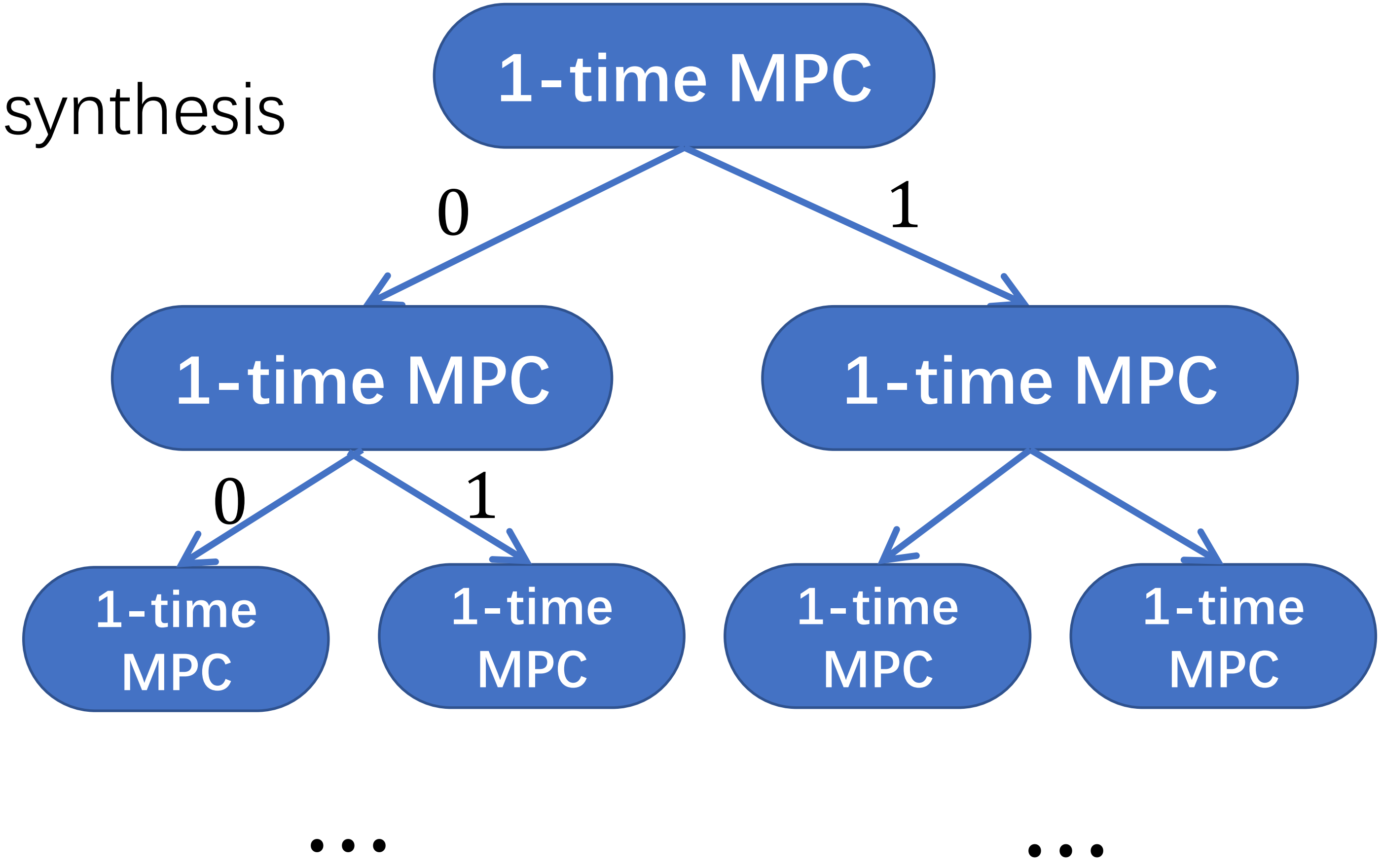

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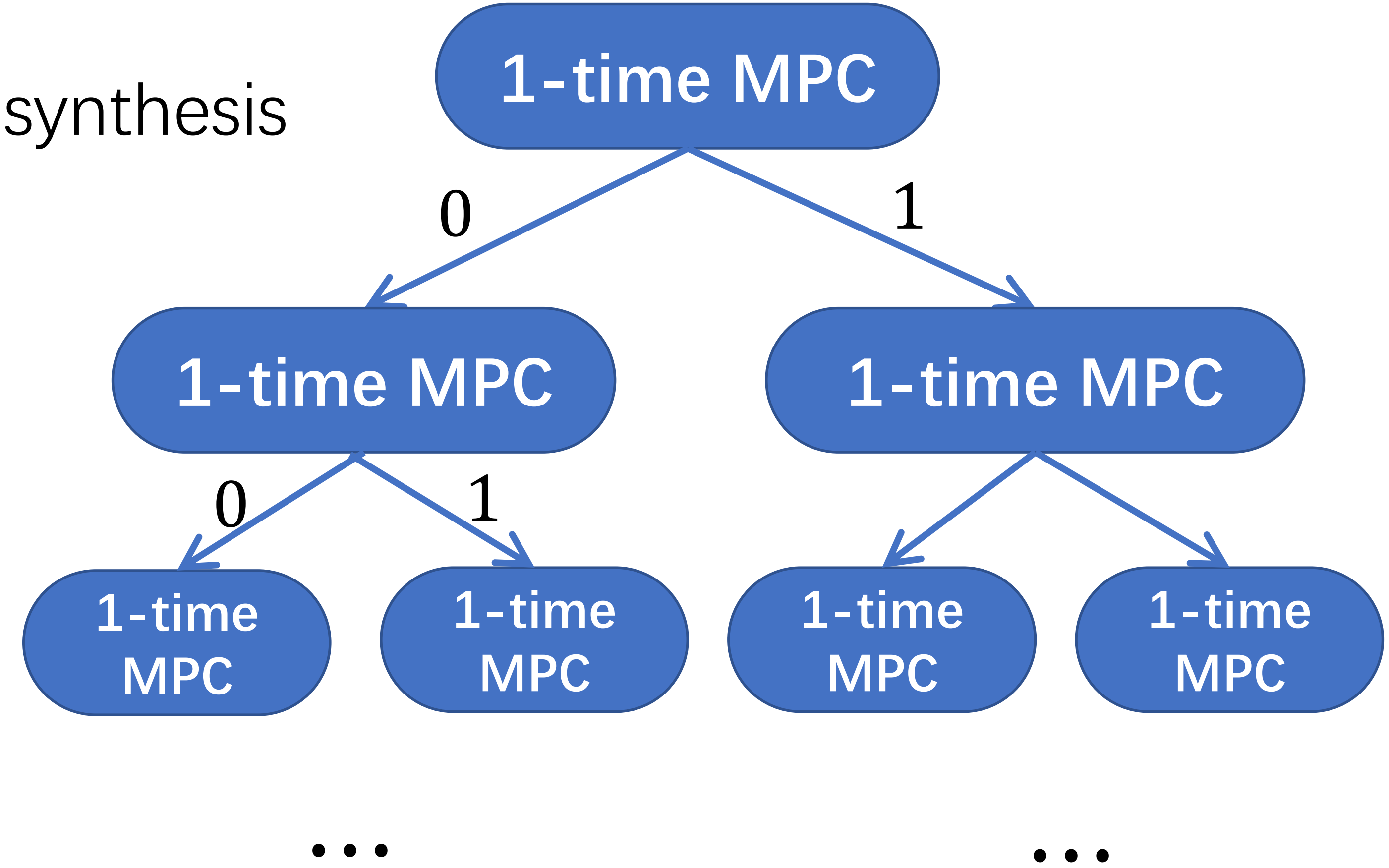

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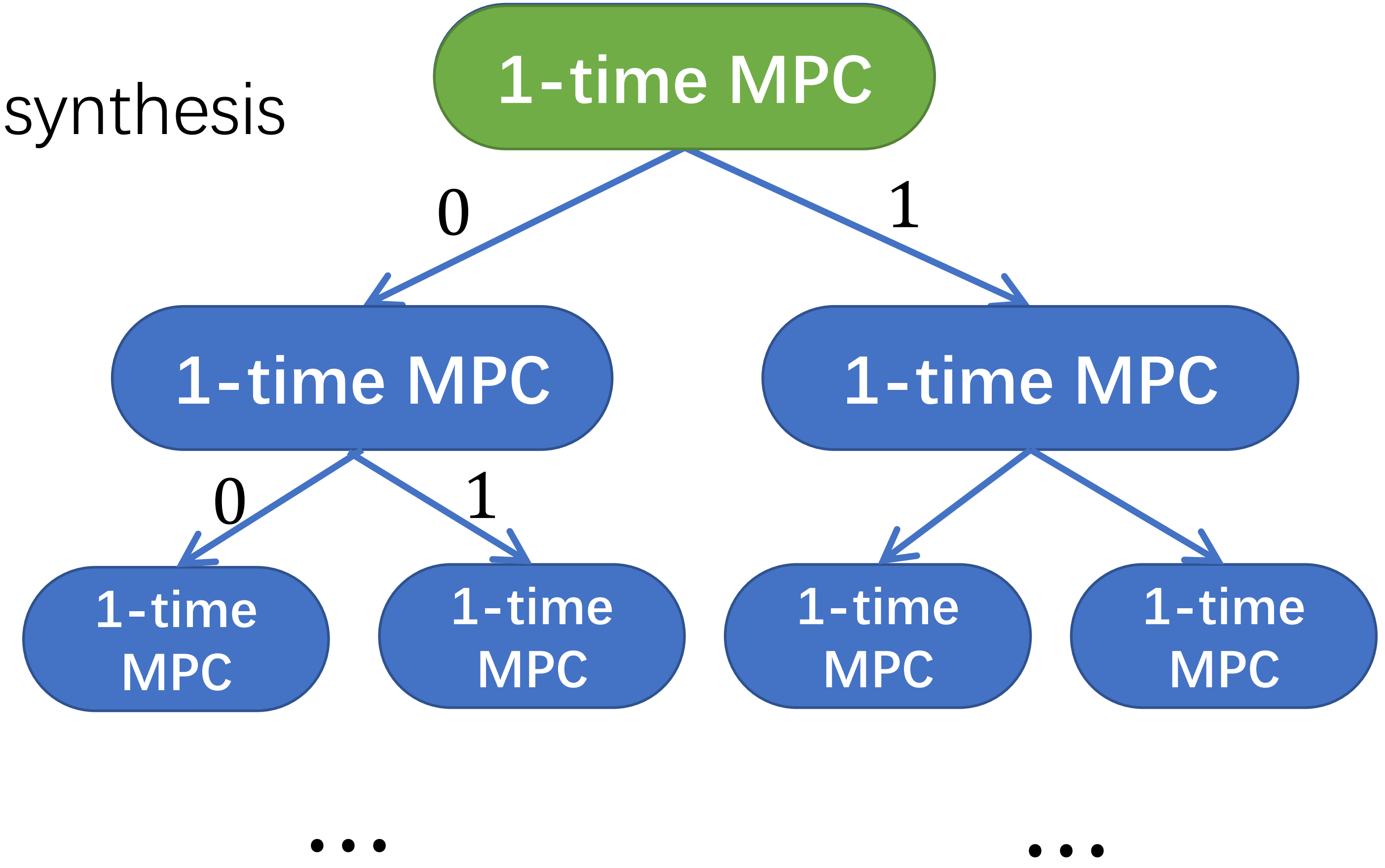

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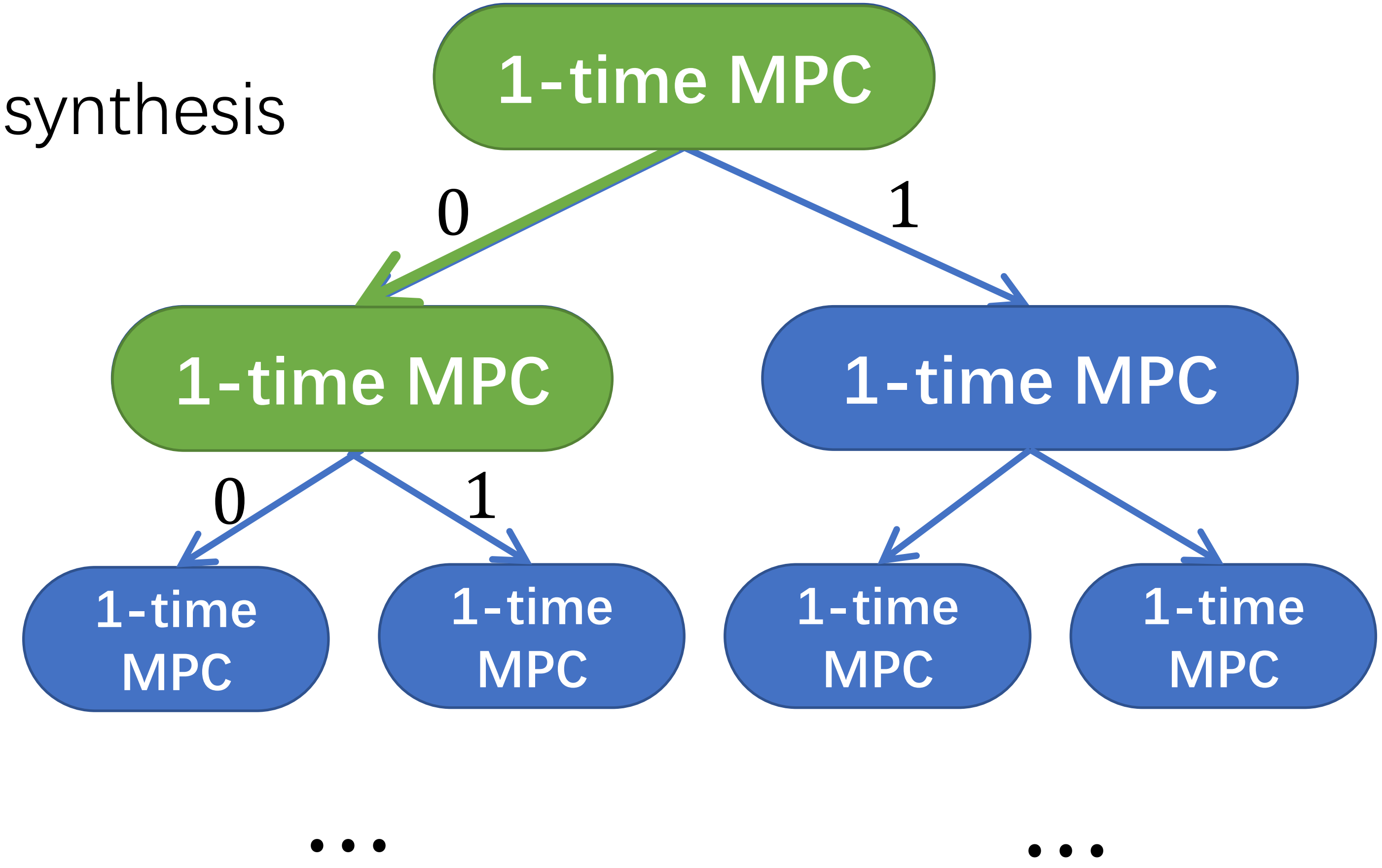

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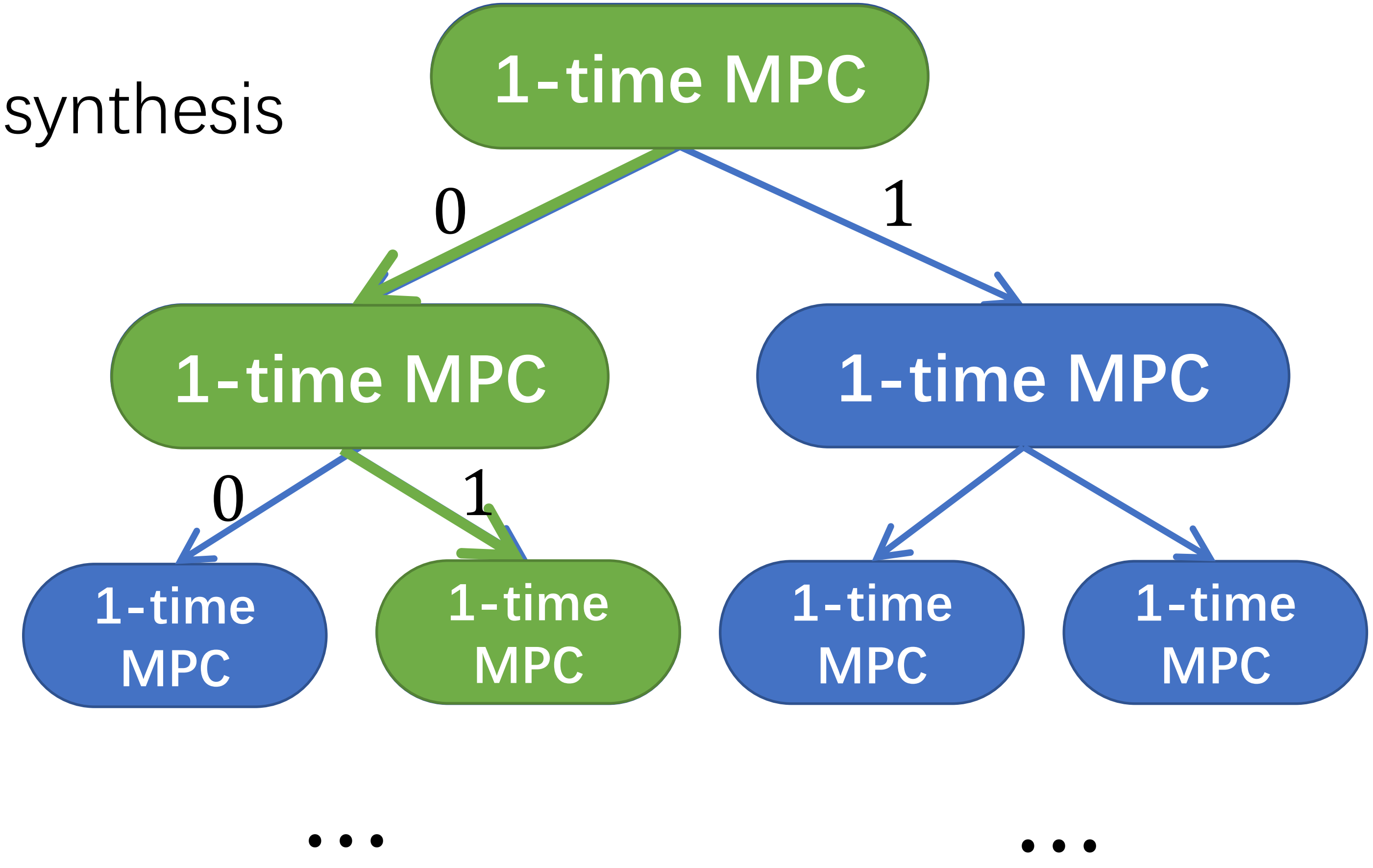

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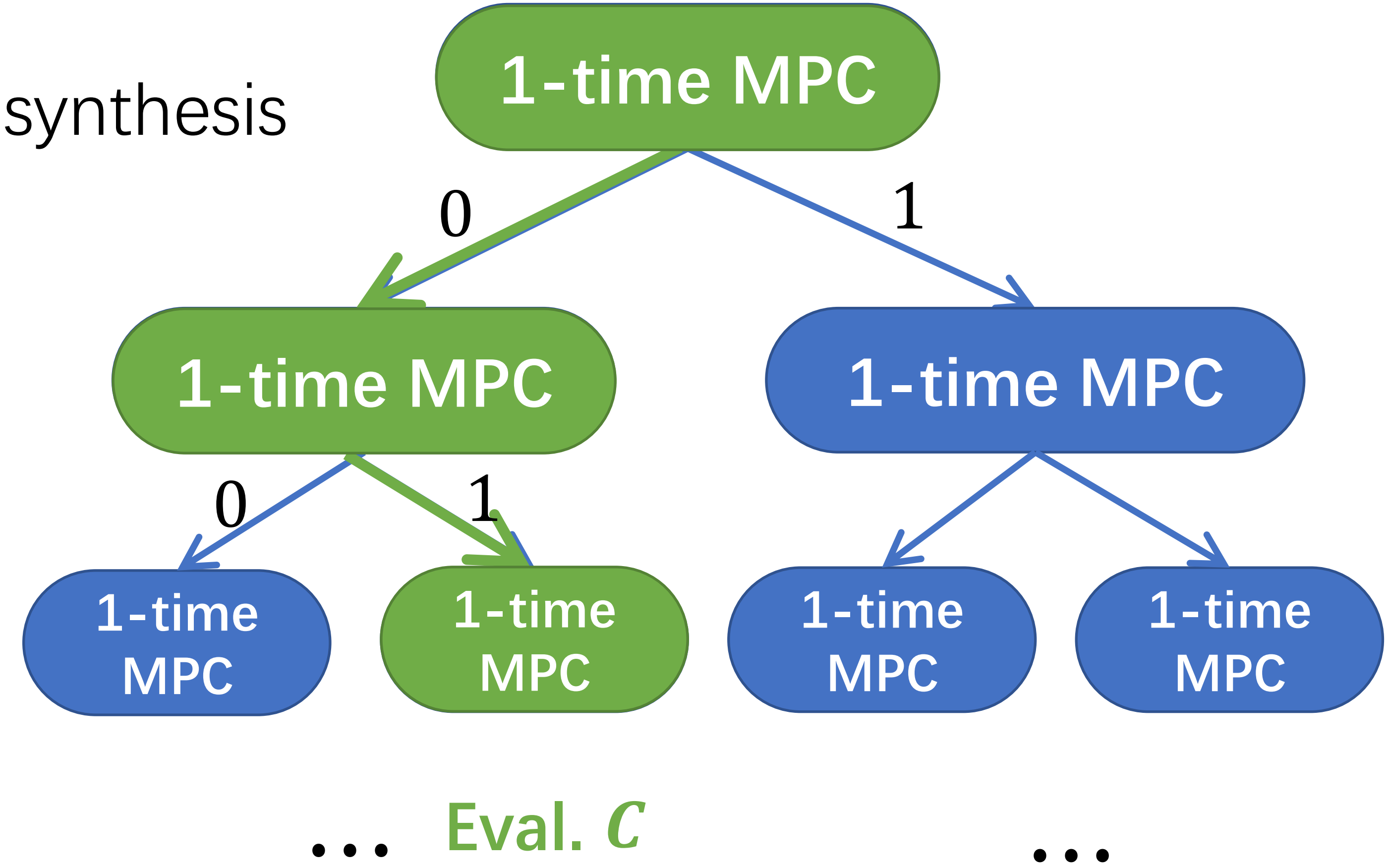
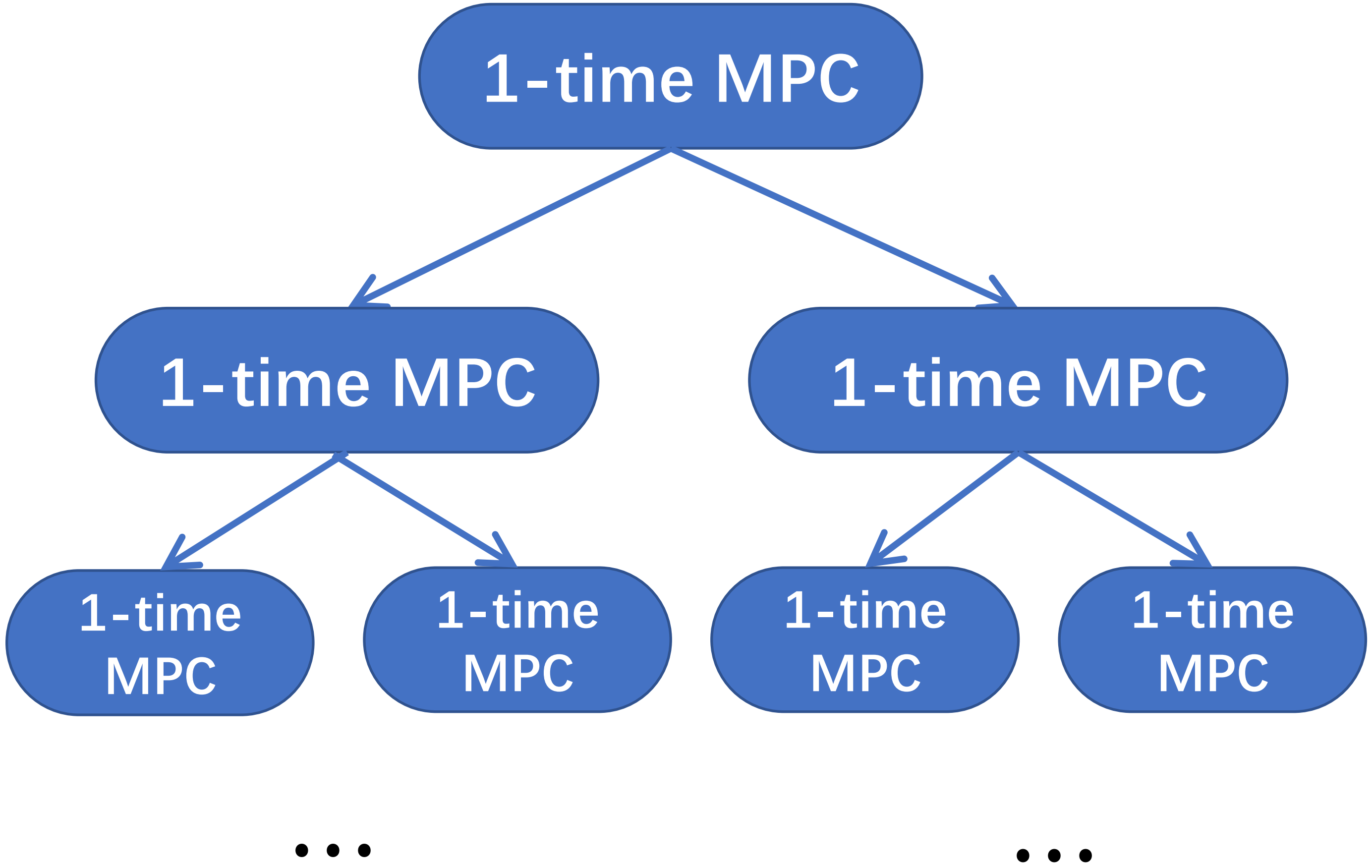


Time Complexity Blow Up



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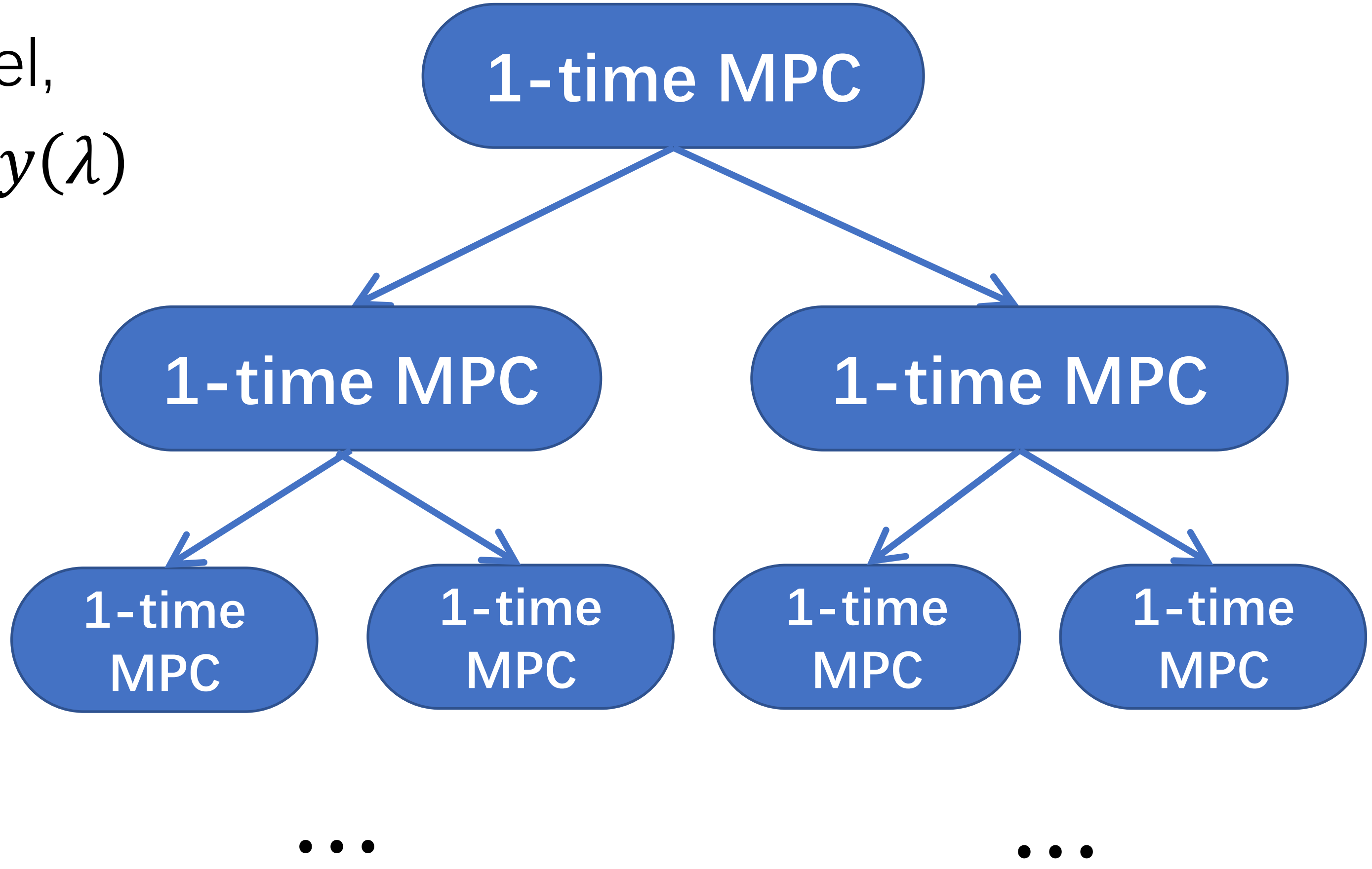

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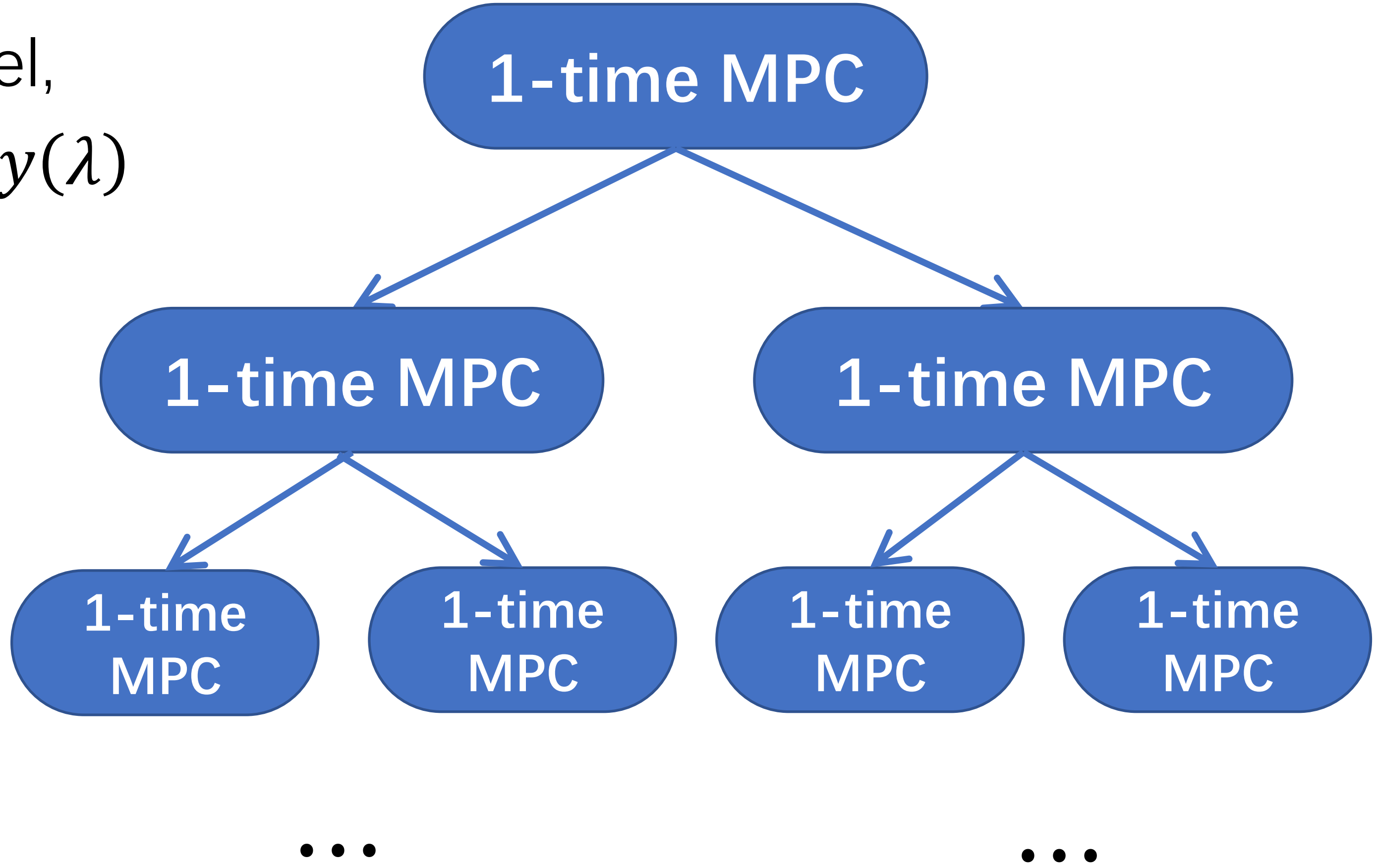

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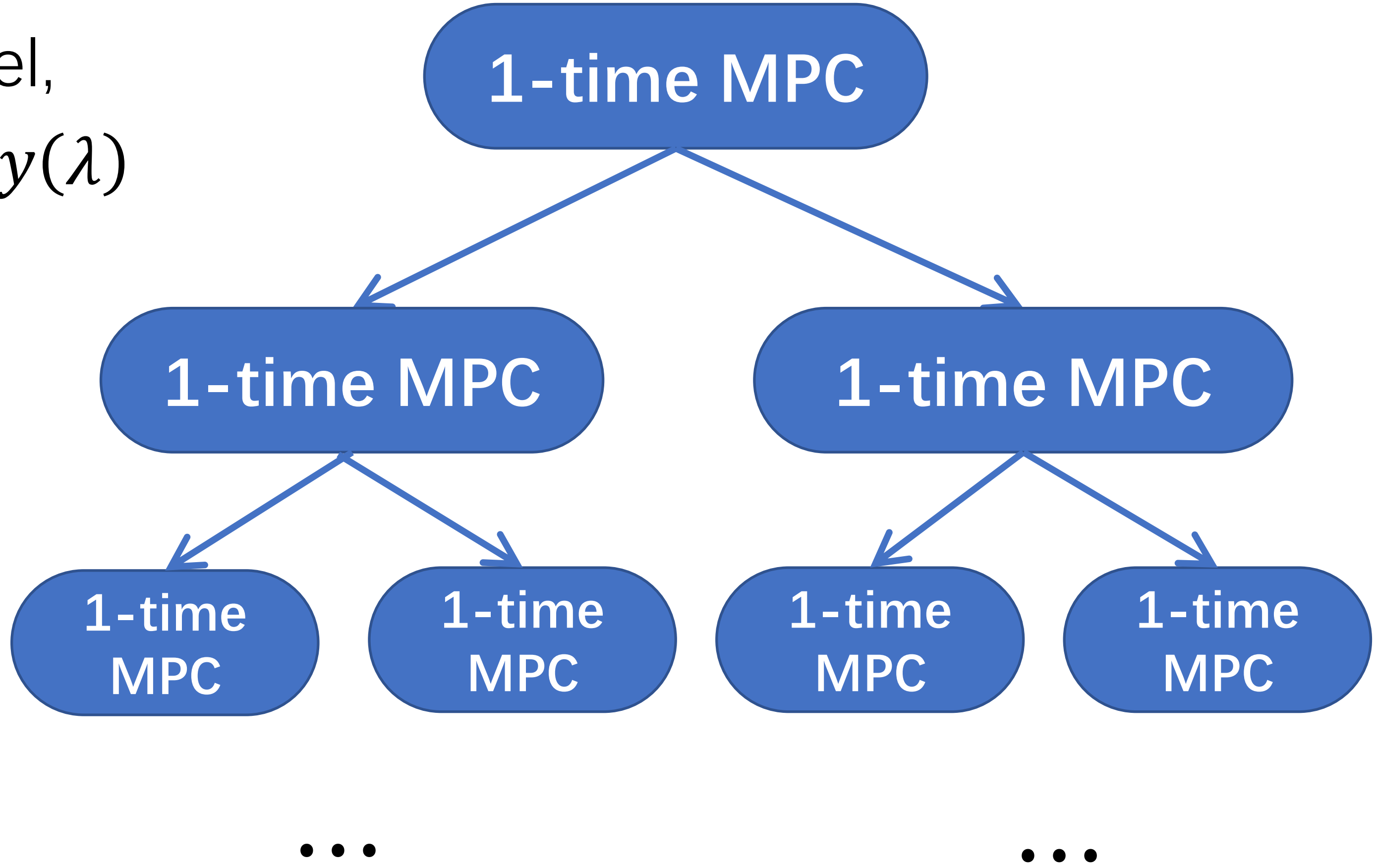

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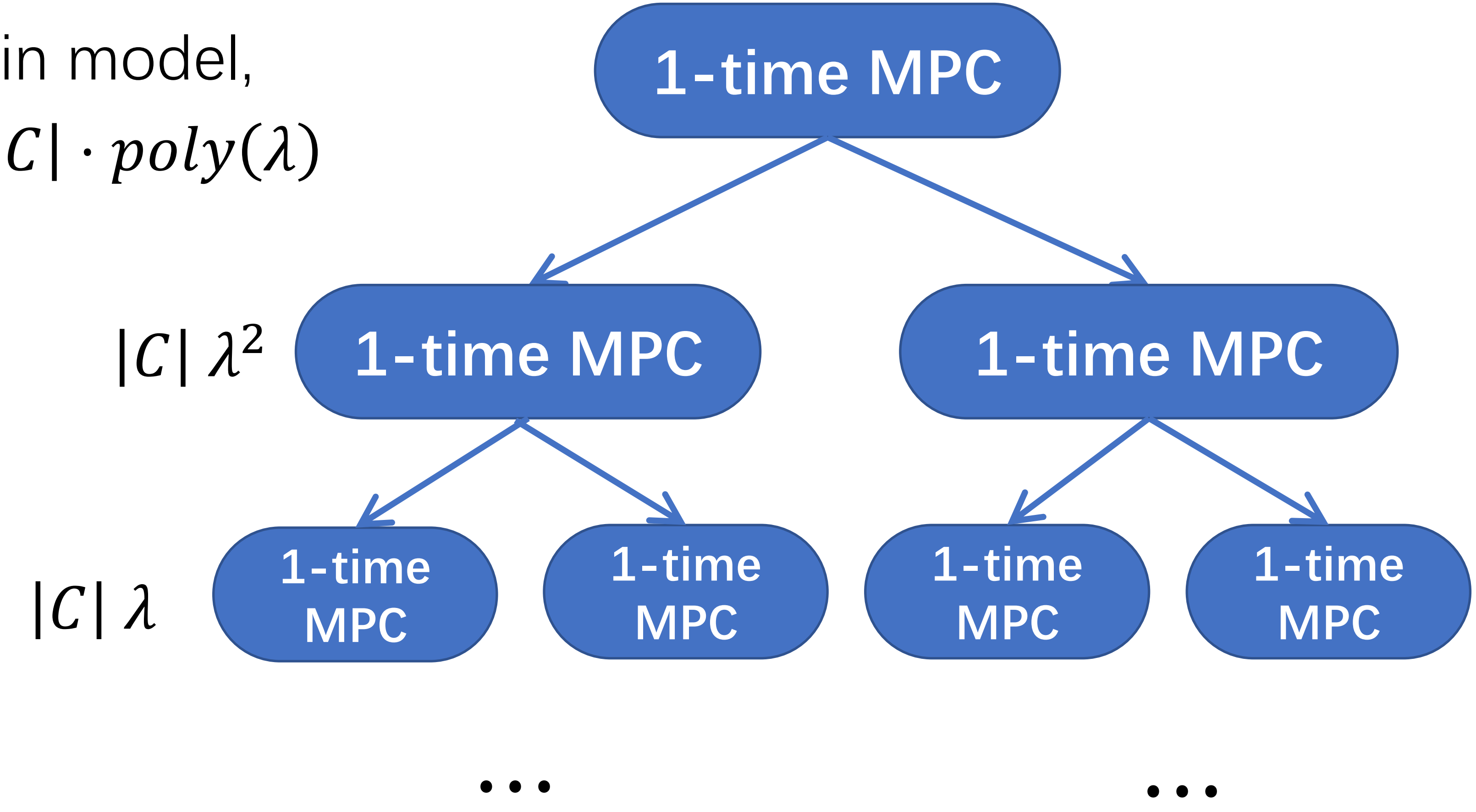

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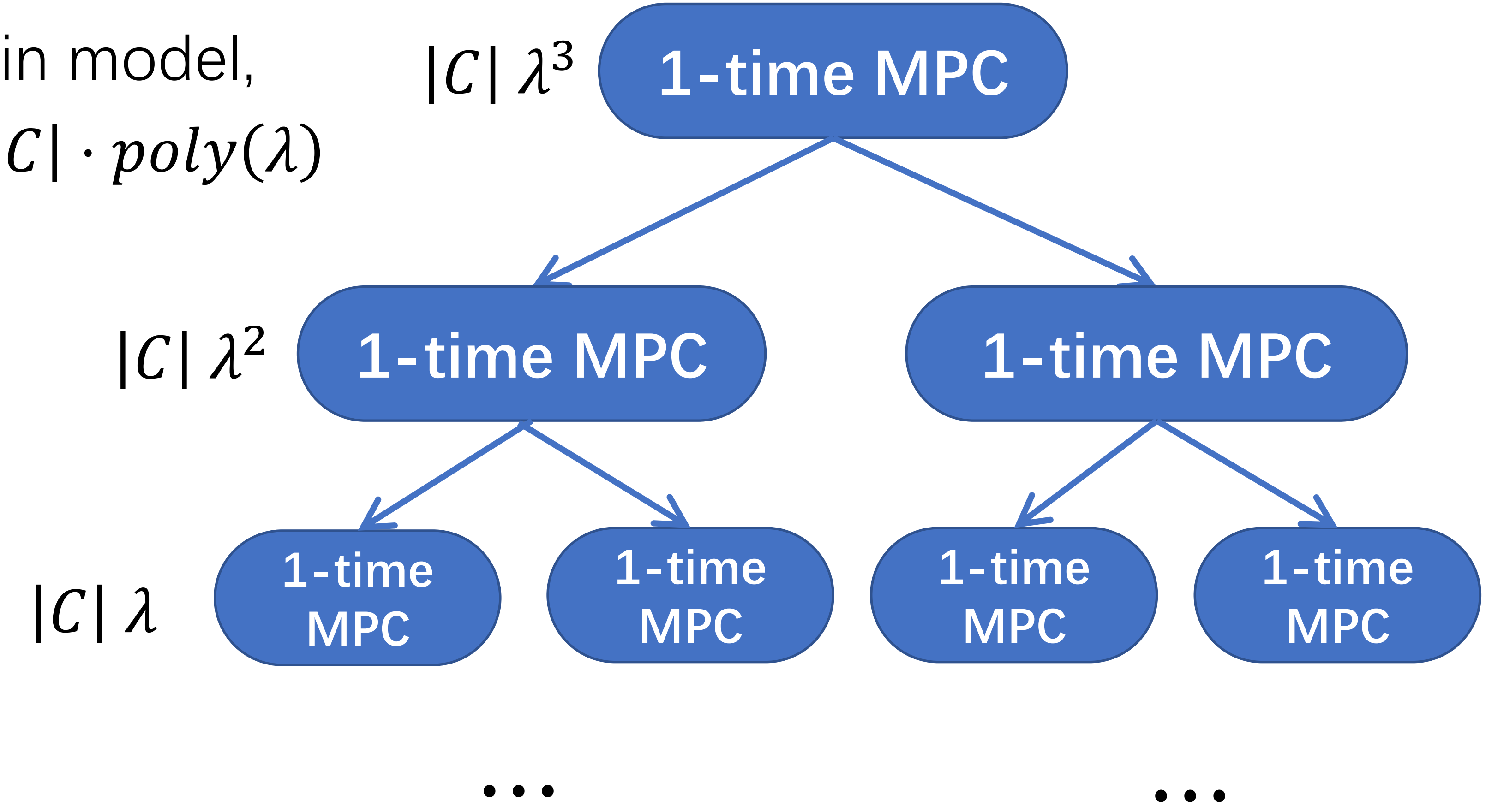

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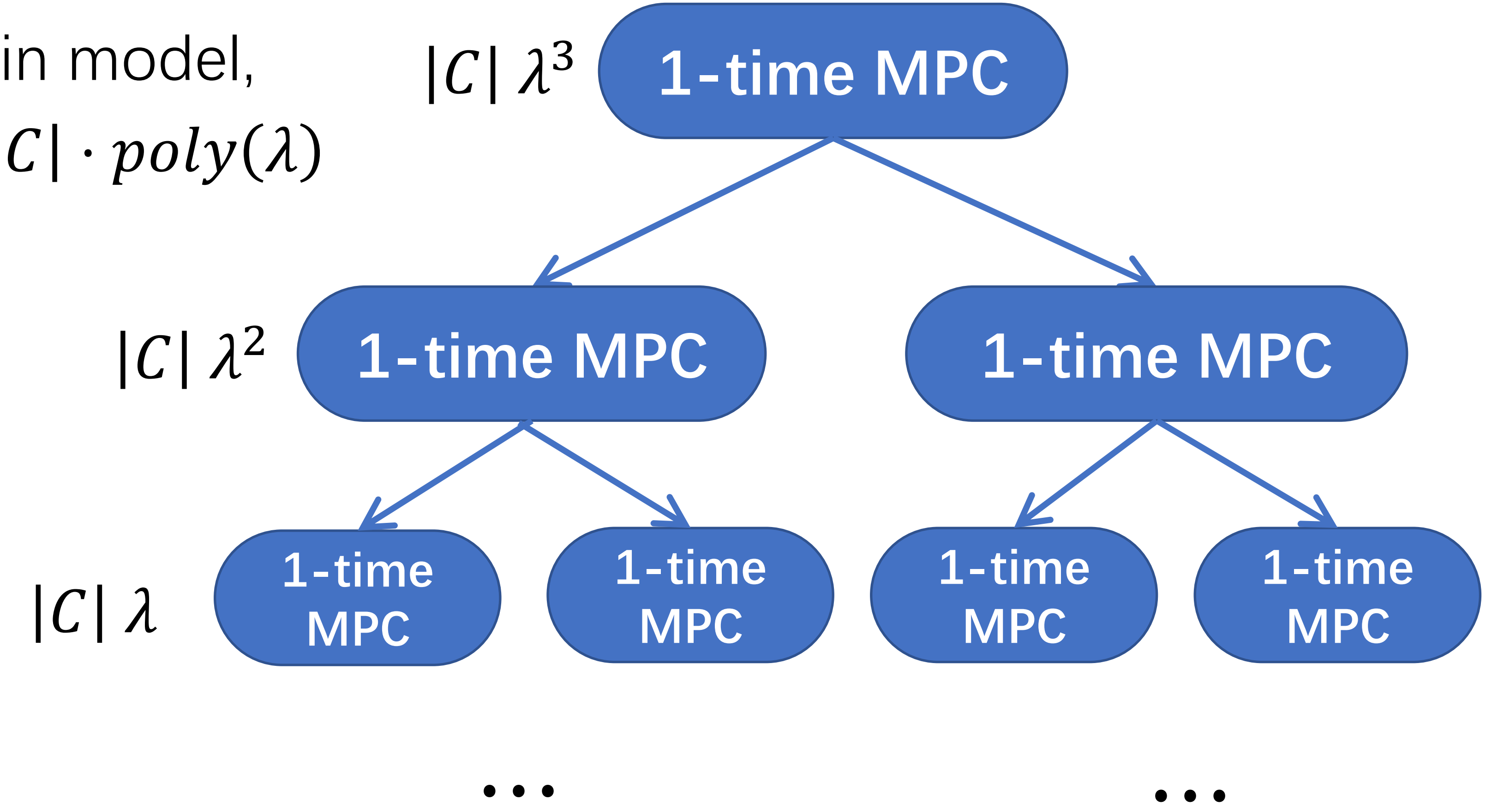
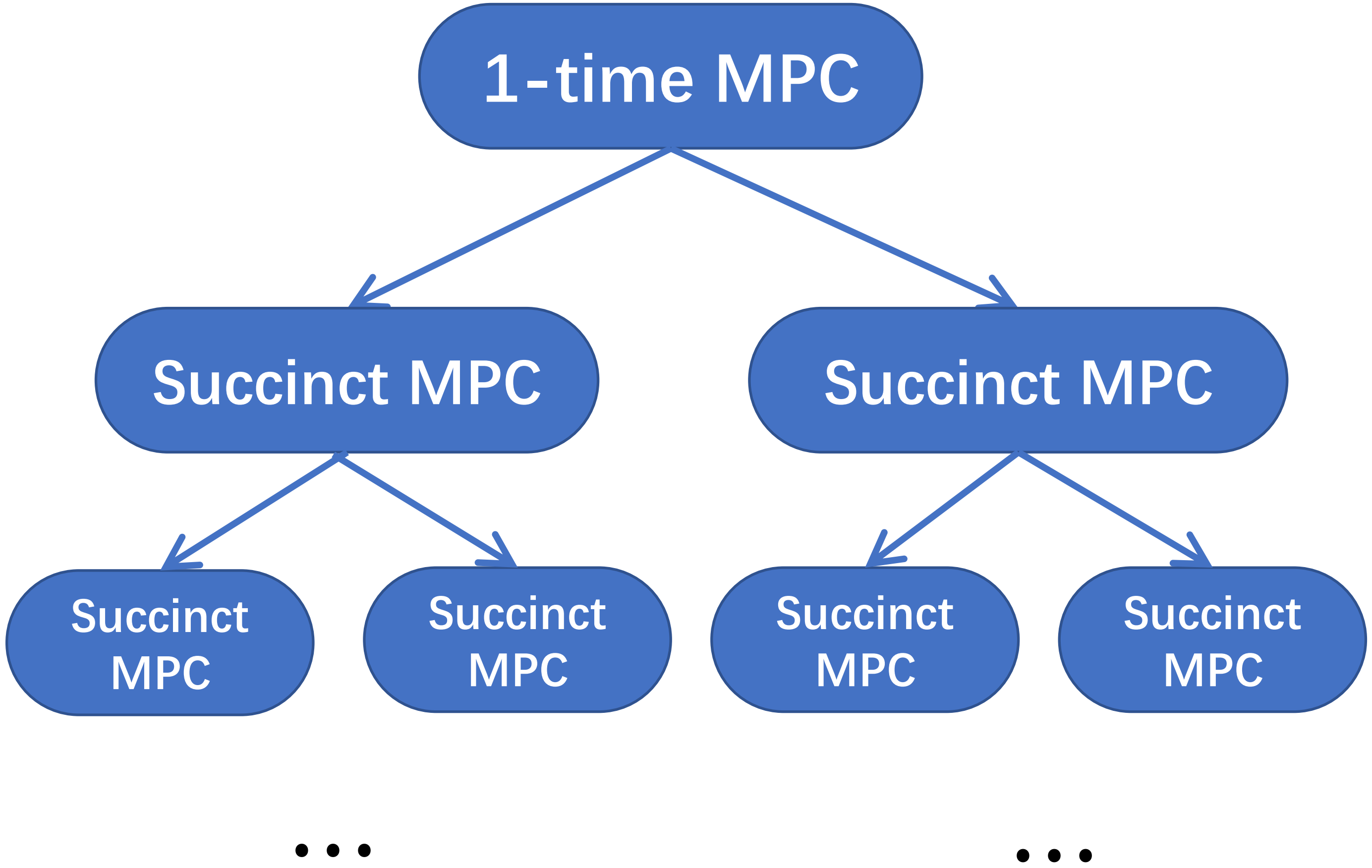
  is **exponential** in λ



Necessary Condition for Recursion



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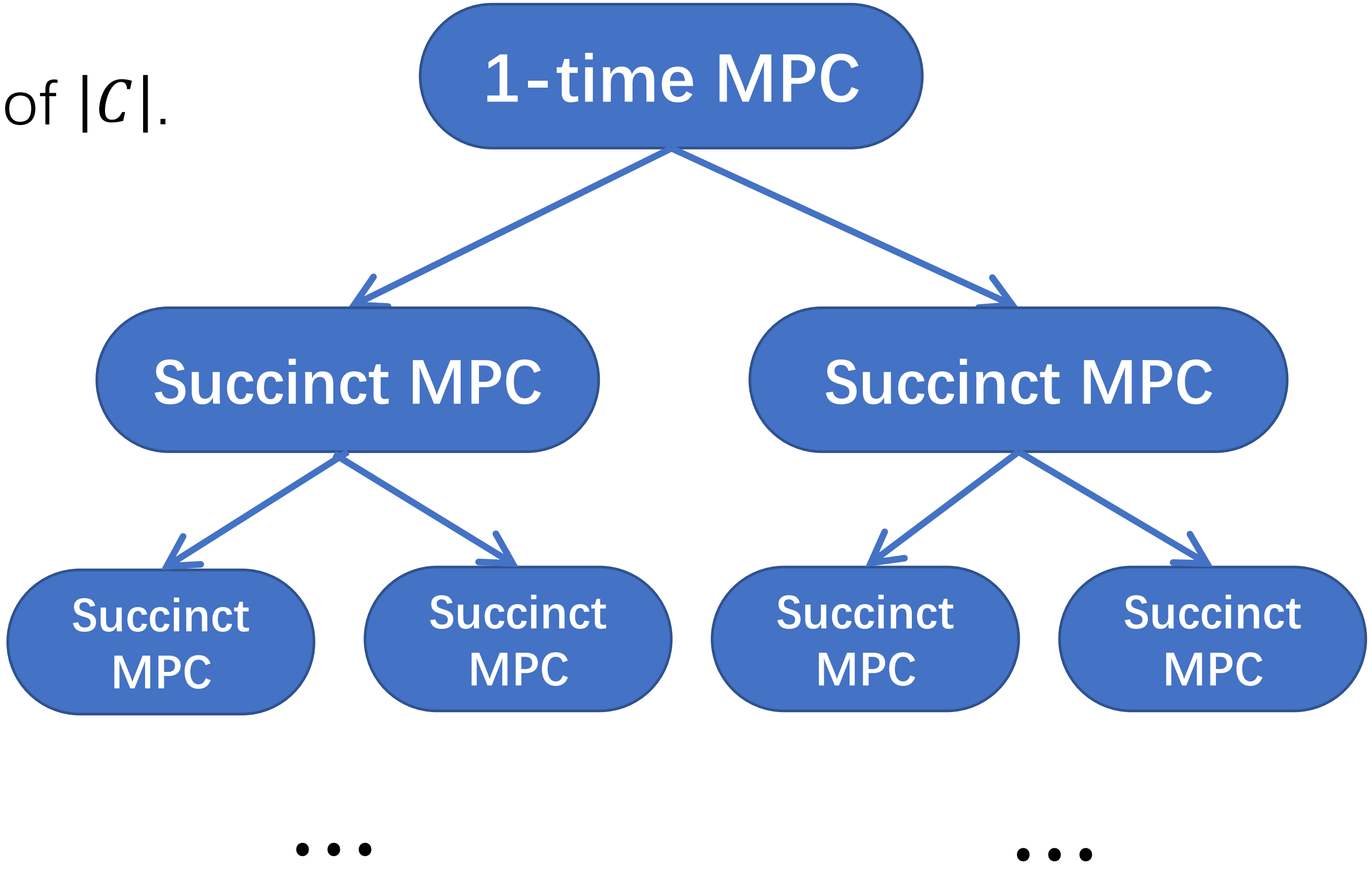

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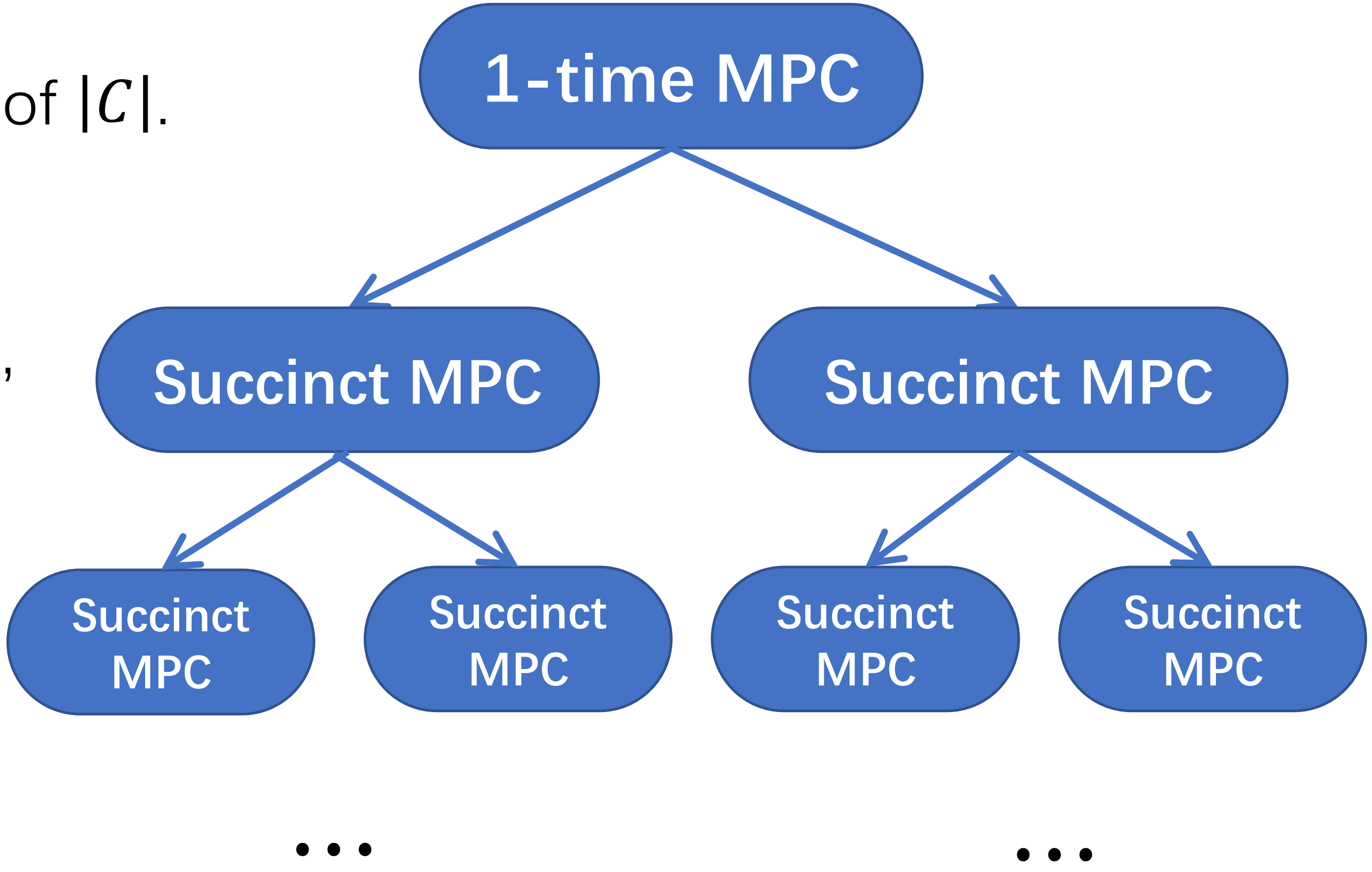

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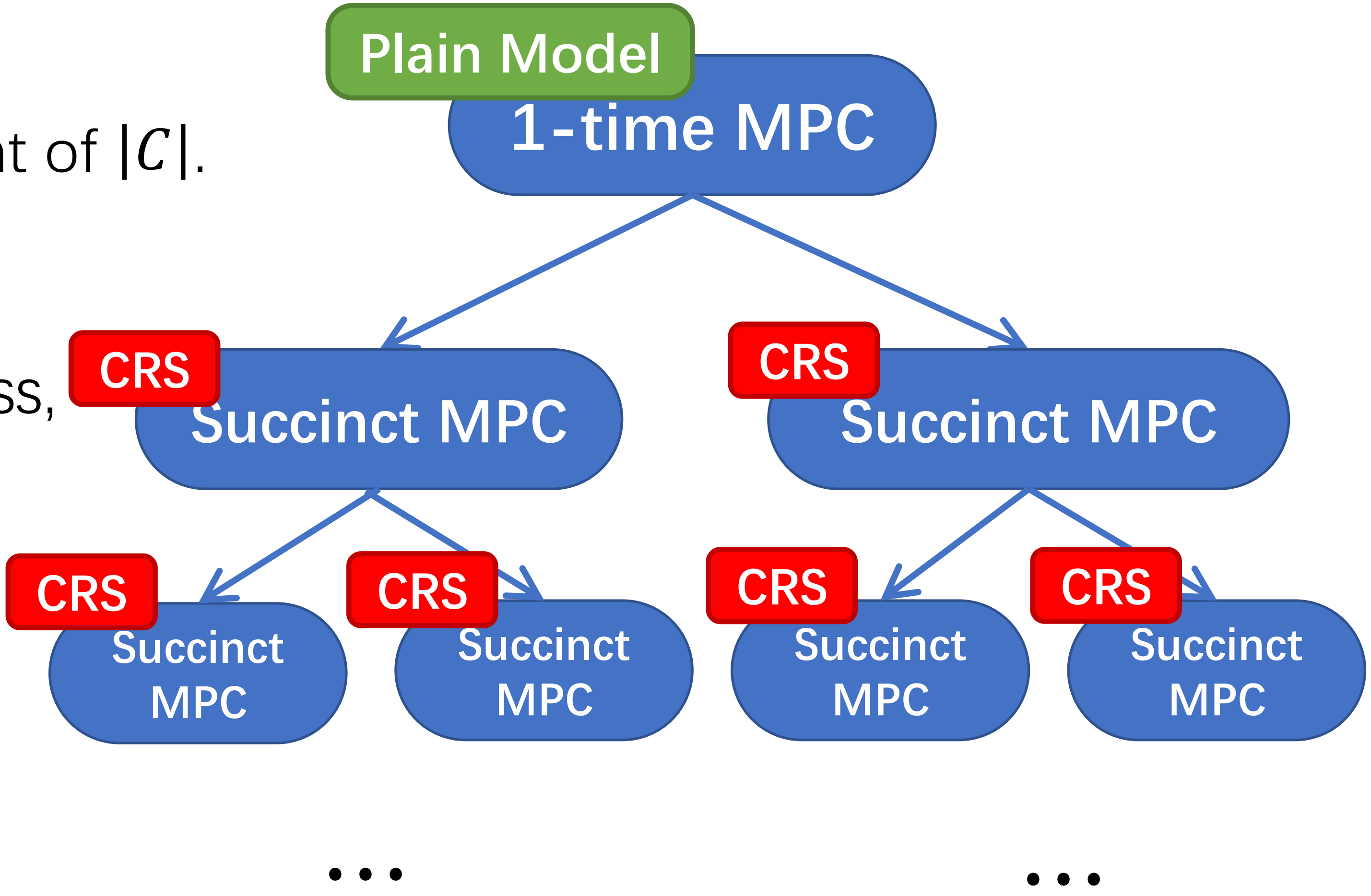

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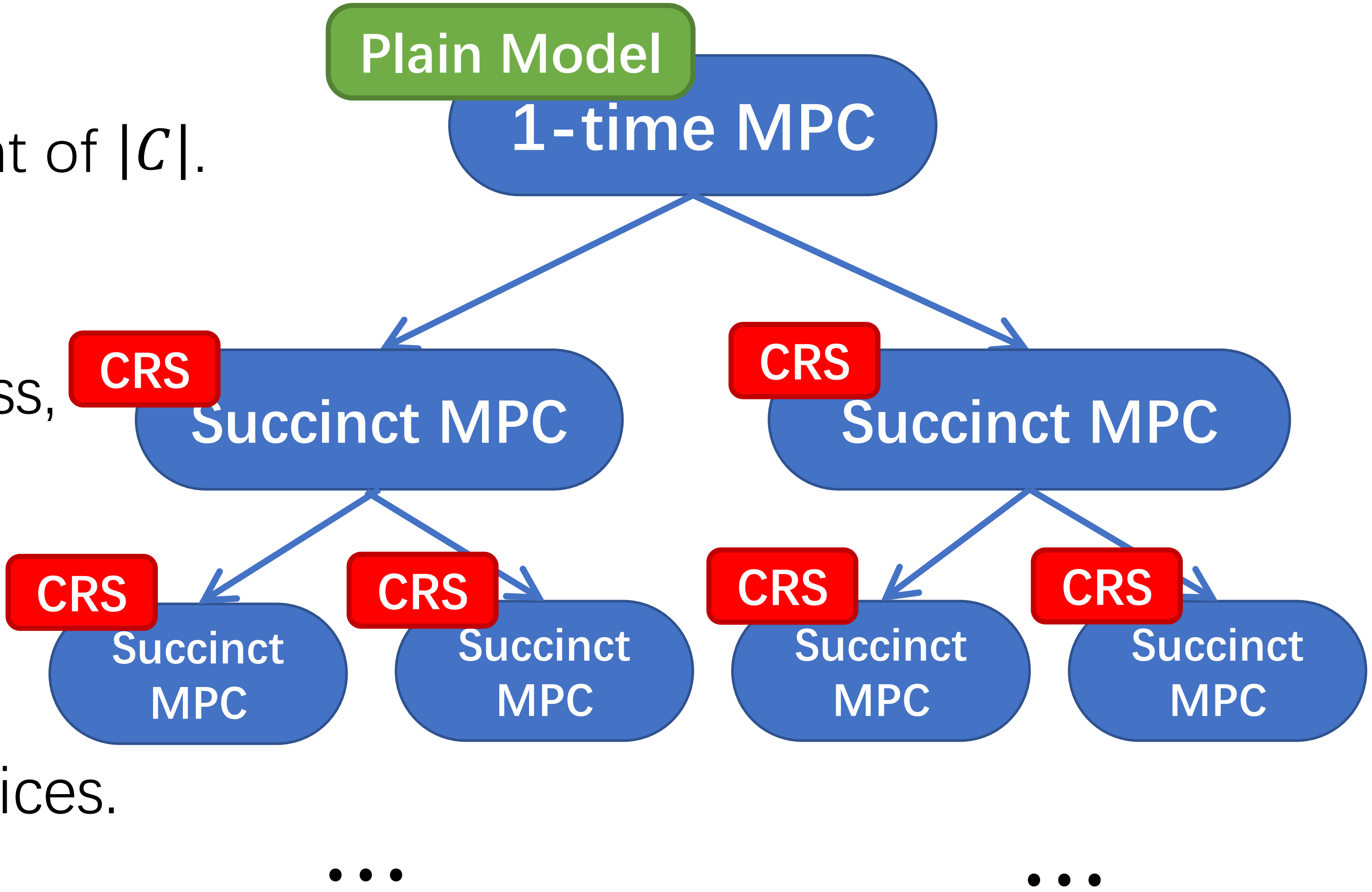
- In fact succinct 1-time MPC in *preprocessing model* suffices.



Thank you!